Calibration of polarization effects for the focusing lens pair in a micro-spot Mueller matrix ellipsometer

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ABSTRACT

With the urgent demands of characterizing the optical properties and thicknesses of nano-films in some micron-scale trenches usually emerging in integrated circuit manufacturing, the micro-spot Mueller matrix ellipsometer (M-MME) has attracted increasingly more attention. While using a lens pair to enable a probing spot with a micron scale in the M-MME, the additional polarization effects caused by the lens pair usually degrade the measurement precision of the instrument. Therefore, it is of great significance to calibrate the polarization effects of the lens pair for measurement precision improvement. In this work, a parametric model based on the Mueller matrix decomposition has been proposed to fully describe the polarization effects of a focusing lens pair in the M-MME. A single lens in the focusing lens pair could be optical equivalent to a cascade system consisting of a circular retarder with slight diattenuation, a linear retarder with small diattenuation, a rotator, and a depolarizer. Correspondingly, a comprehensive calibration method combining an initial offline and a subsequent in-situ calibration was proposed to achieve the ingenious correction of polarization effects for the self-built M-MME. Meanwhile, in order to demonstrate the effectiveness and feasibility of the proposed method, a series of thickness measurement experiments on SiO\textsubscript{2} films have been carried out. The corresponding results indicate that the proposed calibration method could improve the relative deviation between the thickness measured by the self-built M-MME and those measured by the commercial MME to within 1.6% for those SiO\textsubscript{2} films with thickness larger than 25 nm.

1. Introduction

With the steady development of semiconductor industry in accordance with the international roadmap for devices and systems \cite{1,2}, ultra-thin nanofilms and low-dimensional materials have been gradually introduced into the semiconductor devices to improve the photovoltaic performance \cite{3-6}. Due to the relatively small size of the probing area of interest and the high precision needs of measurement, the characterizations of both thickness and optical properties of the low-dimensional materials in the micro-area or the ultrathin nanofilms in some micron-scale trenches are challenging the ability of conventional Mueller matrix ellipsometer (MME) \cite{7-9}. Through utilizing the lens pair to focus and collect the probing beam between the polarization state generation (PSG) and polarization state analysis (PSA) modules in the MME, it is possible to develop a micro-spot MME (M-MME) with a probing spot of less than 20 um for characterizing the thickness and optical properties of the above materials in the micro-area \cite{10-12}. However, the lens pair would inevitably alter the polarization state of the incident beam and the reflective beam \cite{13,14}, which will further affect the measurement precision of the Mueller matrix measured by the instrument. Therefore, it is of great significance to calibrate the polarization effects of the lens pair to improve the measurement precision of the M-MME.

Due to the finite numerical aperture of the focusing lens used in various spectroscopic ellipsometers, the discussions about the polarization effects caused by the focusing lens focus primarily on estimating the depolarization effects \cite{15-17}, in which depolarization occurs by the variation of incidence angle. Although these studies involve complex

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integral calculations, the proposed method is of positive significance for improving the measurement precision in the ellipsometry of nanostructure [16]. Meanwhile, other polarization effects such as the rotating effect of the incidence plane, the optical activity of lens material, the residual strain birefringence caused by the fabrication processes, and the birefringence due to the installation process, have also attracted the attention of both ellipsometry and photo-lithography experts. Correspondingly, several methods based on the Jones matrix [18–21], the Pauli spin matrix [22,23], the orientation Zernike polynomials [24,25], or the Mueller matrix have been proposed to calibrate the polarization effects of the focusing lens pair or the focusing objective [26,27]. Peng et al. used the Jones matrix of a typically isotropic sample to characterize the additional ellipsometric responses caused by an objective, which ensures the measurement performance of their angle-resolved ellipsometer [19]. By decomposing the Jones matrix into a Pauli spin matrix basis to clarify its physical meaning, the polarization aberrations of high numerical aperture projection objectives could be investigated in detail [22]. Meanwhile, Ruoff et al. introduced the orientation Zernike polynomials to assess the Jones pupil for the high numerical aperture lithography lenses, in which the Jones pupil of the projection objective can be decomposed into some scalars, such as wavefront, apodization and rotation, together with two vectorial retardance and diattenuation [25]. However, neither the Jones matrix nor the Pauli spin matrix, nor even the directionnal Zernike polynomials can take the depolarization effects caused by the lens into consideration.

Chen et al. has proposed an in-situ method based on the Mueller matrix to calibrate the polarization effect of an objective lens in MME in a retro-reflection configuration through utilizing a spherical mirror as the reference sample [27]. Although the thickness measurement results of two SiO₂ films in their works verify the necessity and feasibility of the proposed method, the measurement precision seems to need to be further improved for the requirements of thickness measurements of ultra-thin films in the micro-area. Besides, Bian et al. utilized an elliptical retarder model to describe the non-ideal response of an achromatic Fresnel rhomb with minor stress [28,29], which allows the accurate calibration of some fundamental system parameters in the Mueller matrix ellipsometer.

In this work, a phenomenological parametric model based on the Mueller matrix decomposition has been proposed to fully describe the polarization effects of individual lenses in the used lens pair in the M-MME. In the parametric model, the single lens could be optically equivalent to a cascade system consisting of a linear retarder with slight diattenuation, a linear retarder with small diattenuation, a rotator and a depolarizer. Correspondingly, by using six parameters, including circular phase retardance and diattenuation angles, linear phase retardance and diattenuation angles, and two depolarization factors as the retrieving parameters, the polarization effects of the single lens can be fully modeled. Meanwhile, a comprehensive calibration method combining an initial offline and a subsequent in-situ calibration was proposed to achieve the ingenious correction of polarization effects for the lens pair in the self-built M-MME. Then, in order to demonstrate the effectiveness and feasibility of the proposed method, a series of thickness measurement experiments on SiO₂ films have been carried out, and the corresponding results will be discussed and analyzed in detail.

2. Characterization of the lens pair

As shown in Fig. 1(a), through inserting a lens pair into the light path between the PSG and PSA modules, the conventional MME can be transformed to the M-MME with a focusing probe spot. Meanwhile, the propagation of probe beam through the optical interfaces, the coating layer and the transparent medium of the lens pair can lead to some additional polarization-dependent behaviors, such as diattenuation, retardance and depolarization [30]. These additional polarization effects usually disturb the Mueller matrix of the measured samples, which will finally degrade the measurement precision of the instrument without correction of the polarization effects. Thus, in order to guarantee the measurement precision of the M-MME, it is essential to calibrate the polarization effects of the lens pair as accurately as possible. Since both the focusing lens and the collection lens in the used lens pair are made of the same materials and by the same process, the polarization effects of the two lenses in the lens pair can be described by the same modeling method. According to the phenomenological polarization effects, a parametric model based on the Mueller matrix decomposition could be proposed to fully describe the polarization effects of individual lenses in the used lens pair. In the parametric model, the single lens is optically equivalent to a cascade system consisting of a circular retarder with slight diattenuation, a linear retarder with small diattenuation, a rotator and a depolarizer, as shown in Fig. 1(b). Then, the corresponding parametric model is shown in the following formula,

\[
M_{\text{Lens}} = M_{\text{Dep}}(\varepsilon, \rho) \cdot M_{\text{Rot}}(\delta_{1}/2) \cdot M_{\text{LDLB}}(\phi_{1}, \delta_{1}) \cdot M_{\text{CDCB}}(\phi_{2}, \delta_{2}),
\]

where the Mueller matrix \(M_{\text{Lens}}\) of a single lens in the lens pair is decomposed into the orderly multiplication of the Mueller matrices \(M_{\text{CDCB}}, M_{\text{LDLB}}, M_{\text{Rot}},\) and \(M_{\text{Dep}}\). \(M_{\text{CDCB}}\) and \(M_{\text{LDLB}}\) are the Mueller matrices of a circular retarder with small diattenuation, a linear retarder with slight diattenuation [31–33], respectively. \(M_{\text{Rot}}\) and \(M_{\text{Dep}}\) are the Mueller matrices of an optical rotator and a depolarizer, respectively [34–36]. Both the parameters \(\varepsilon\) and \(\rho\) are the depolarization-related coefficients. The parameters \(\phi_{1}\) and \(\delta_{1}\) represent the linear diattenuation and the linear phase retardance, respectively. The parameters \(\phi_{2}\) and \(\delta_{2}\) are the circular diattenuation and the circular phase retardance, respectively. The optical rotatory angle of the optical rotator is \(-\delta_{2}/2\).

More specifically, these Mueller matrices \(M_{\text{CDCB}}, M_{\text{LDLB}}, M_{\text{Rot}},\) and \(M_{\text{Dep}}\) have the following forms,

\[
M_{\text{CDCB}}(\phi_{2}, \delta_{2}) = \begin{bmatrix}
1 & 0 & 0 & -\cos(2\phi_{2}) \\
0 & \sin(2\phi_{2})\cos(\delta_{2}) & \sin(2\phi_{2})\sin(\delta_{2}) & 0 \\
0 & -\sin(2\phi_{2})\sin(\delta_{2}) & \sin(2\phi_{2})\cos(\delta_{2}) & 0 \\
-\cos(2\phi_{2}) & 0 & 0 & 1
\end{bmatrix}
\]
3. Calibration method and experiments

In order to calibrate the polarization effects of the lens in the focusing lens pair used in the self-built M-MME, a comprehensive calibration method combining an initial offline and a subsequent in-situ calibration has been proposed. The off-line calibration was carried out based on a commercial spectral MME (RC2, J. A. Woollam, USA), while the in-situ calibration was implemented based on a self-built M-MME. Moreover, the calibration was implemented based on a self-built M-MME. Moreover, the calibration method was shown in Fig. 2.

Fig. 2(a) shows the offline calibration procedure, where the focusing lens was mounted on the self-built holding apparatus on the sample stage of the M-MME. Under the direct-transmission measurement mode of the commercial MME, the Mueller matrices of the air and the focusing lens pair were sequentially measured by the instrument. Then, the Mueller matrix of the lens pair used in the self-built M-MME, a comprehensive calibration of the two lenses can be considered the same under the premise of both the same materials and the same process. By fitting the measured Mueller matrices of the single lens with the parameterized Mueller matrix model, seven polarization parameters of interest can be achieved. Due to the small numerical aperture of the lens and the beam diameter much smaller than the focal length [27], the polarization effect caused by the lens is small. Thus, the polarization effect parameters of the air medium can be considered as the fitting initial values of the proposed parametric model. The fitting process was essentially an inverse optimization problem [37], which could be described in the following form,

\[
\hat{x} = \arg \min_{x \in \Omega} \sum_{\lambda_k} [M_{\text{Lens-exp}}(x, \lambda_k) - M_{\text{Lens-cal}}(x, \lambda_k)]^T \Gamma_\lambda(\lambda_k) [M_{\text{Lens-exp}}(x, \lambda_k) - M_{\text{Lens-cal}}(x, \lambda_k)],
\]

where \(M_{\text{Lens-cal}}(x, \lambda_k) = M_{\text{Lens}}(p_1, \delta_1, p_2, \delta_2, e, p, \lambda_k)\) is the theoretical Mueller matrix of the lens calculated by using the proposed parametric model at the wavelength \(\lambda_k\). The vector \(x = (p_1, \delta_1, p_2, \delta_2, e, p)\) represents a set of six polarization effect parameters. \(\Gamma_\lambda(\lambda_k)\) is the Moore-Penrose pseudo-inverse of the covariance matrix of the measured Mueller matrix \(M_{\text{Lens-exp}}\) at the wavelength \(\lambda_k\) [38]. \(\Omega\) represents the feasible domain of the vector \(x\).

Furthermore, by characterizing the Mueller matrices and the corresponding polarization parameters of at least 29 lens pairs with the same nominal specifications, it is easy to screen out the predominant polarization effect parameters from the six parameters by means of careful statistical analysis. These predominant polarization effect parameters will govern the variation of the single lens’s Mueller matrix. The above method of reducing the number of polarization effect parameters will likely significantly suppress the degree of overfitting in subsequent in-situ calibrations.

The in-situ calibration experiments were carried out based on the self-built micro-spot Mueller matrix ellipsometer with single wavelength of 632.8 nm, as shown in Fig. 2(b). The focusing lens pair (customized L1 & L2, Dongfliai Optics&Electronics Enterprise, China) was used to focus and collect the probing beam between the PSG and the PSA modules. As the instrument switches from the large-spot measurement mode to the micro-spot measurement mode, the diameter of the probe spot will be reduced from 1.89 mm to 14 um by using the lens pair. Details related to the development of the self-built M-MME are omitted here for the sake of brevity. One can consult the Ref. [10] for more details. The whole in-situ calibration process includes the following two steps. Under the theoretical Mueller matrix of air is an identity matrix, but its actual measured Mueller matrix has non-zero off-diagonal elements with values less than 0.004. Moreover, through assuming that the constituent materials of the two lenses are reciprocal [27], the Mueller matrices of the two lenses can be considered the same under the premise of both the same materials and the same process. By fitting the measured Mueller matrix \(M_{\text{Lens-exp}}\) of the single lens with the parameterized Mueller matrix model, seven polarization parameters of interest can be achieved. Due to the small numerical aperture of the lens and the beam diameter much smaller than the focal length [27], the polarization effect caused by the lens is small. Thus, the polarization effect parameters of the air medium can be considered as the fitting initial values of the proposed parametric model. The fitting process was essentially an inverse optimization problem [37], which could be described in the following form,

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\]

where \(M_{\text{Lens-cal}}(x, \lambda_k) = M_{\text{Lens}}(p_1, \delta_1, p_2, \delta_2, e, p, \lambda_k)\) is the theoretical Mueller matrix of the lens calculated by using the proposed parametric model at the wavelength \(\lambda_k\). The vector \(x = (p_1, \delta_1, p_2, \delta_2, e, p)\) represents a set of six polarization effect parameters. \(\Gamma_\lambda(\lambda_k)\) is the Moore-Penrose pseudo-inverse of the covariance matrix of the measured Mueller matrix \(M_{\text{Lens-exp}}\) at the wavelength \(\lambda_k\) [38]. \(\Omega\) represents the feasible domain of the vector \(x\).

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\[
\hat{x} = \arg \min_{x \in \Omega} \sum_{\lambda_k} [M_{\text{Lens-exp}} - M_{\text{Lens-cal}}(x, \lambda_k)]^T \Gamma_\lambda(\lambda_k) [M_{\text{Lens-exp}} - M_{\text{Lens-cal}}(x, \lambda_k)],
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where \(M_{\text{Lens-cal}}(x, \lambda_k) = M_{\text{Lens}}(p_1, \delta_1, p_2, \delta_2, e, p, \lambda_k)\) is the theoretical Mueller matrix of the lens calculated by using the proposed parametric model at the wavelength \(\lambda_k\). The vector \(x = (p_1, \delta_1, p_2, \delta_2, e, p)\) represents a set of six polarization effect parameters. \(\Gamma_\lambda(\lambda_k)\) is the Moore-Penrose pseudo-inverse of the covariance matrix of the measured Mueller matrix \(M_{\text{Lens-exp}}\) at the wavelength \(\lambda_k\) [38]. \(\Omega\) represents the feasible domain of the vector \(x\).
large-spot measurement mode shown as the red spot area in Fig. 2(e), a standard SiO$_2$ film sample (SiO$_2$ film with known thickness, Wuhan Eoptics Technology Co. Ltd., China) was first measured and then analyzed to determine the systematic parameters of the self-built M-MME. These systematic parameters include the polarizer-, the compensator-, and the detector-related polarization parameters. Under the micro-spot measurement mode as shown in Fig. 2(c)–(e), the standard SiO$_2$ film sample would be measured again. Then, both the SiO$_2$ film thickness and the polarization effect parameters of the single lens were determined by fitting the measured light intensity. The corresponding fitting equation was shown in the following formula,

$$\hat{y} = \arg\min_{y\in\Omega} |I_{\text{meas}} - I_{\text{calc}}(y)|^T \Gamma_I^{-1} |I_{\text{meas}} - I_{\text{calc}}(y)|,$$

where $I_{\text{meas}}$ and $I_{\text{calc}}(y)$ are the measured and the theoretically calculated intensity matrices, respectively. $\Gamma_I$ is the Moore-Penrose pseudo-inverse of the covariance matrix of the measured intensity matrix. The vector $y = [d, \phi_1, \delta_1, \phi_2, \delta_2, e, p]$ represents a set of the retrieval parameters. $\Omega_y$ represents the feasible domain of the vector $y$.

4. Results and discussions

4.1. Offline calibration of the lens pair

Fig. 3 shows the fitting results in the spectral range of 632–633.5 nm for a representative lens reported from the offline calibration experiments. Three theoretical models are used to fitting the measurement Mueller matrix, and the corresponding fitting results are represented in Fig. 3(a). These three models are the proposed parametric model, an equivalent retarder with a rotatory angle [39], and the standard Lu-Chipman combination of a depolarizer, a retarder, and a diattenuator [32], respectively. According to the fitting analysis based on the proposed parametric model, the overall fitting deviation of each Mueller matrix element is less than 0.002 in the spectral range, and the fitting deviation of each Mueller matrix element at 632.8 nm is less than 0.0003. Moreover, the highly consistent fitting of the off-diagonal elements of the Mueller matrix indicates that the proposed model can accurately reveal both the anisotropy and the depolarization characteristics of the focusing lens. While the other two models are difficult to match the variation trend of the off-diagonal elements of the Mueller matrix measured in the interesting spectral range. By precisely calibrating the off-diagonal elements of the lens’s Mueller matrix, it can minimize the residual polarization artifacts in the measured Mueller matrix of the sample caused by the instrument. As for the main diagonal elements in the Mueller matrix, the fitting consistency between the proposed model and the measurement results is significantly higher than that based on the other two models. These results indicate that the proposed model can objectively reveal the diattenuation and the isotropic characteristics of the focusing lens.

Correspondingly, the six polarization effect parameters retrieved by the fitting analysis and the fitting error are shown in Fig. 3(b). The fitting error based on the sum of mean square errors of each element in the Mueller matrix is generally less than $1.3 \times 10^{-5}$, and specifically the fitting error at 632.8 nm is less than $1.56 \times 10^{-6}$. These results indicate the acceptable goodness-of-fitting. According to the results shown in Fig. 3(b), as the wavelength varies from 632 nm to 633.5 nm, the linear phase retardance $\delta_1$ and the circular retardance $\delta_2$ exhibit strong oscillations near their respective mean values. Specifically speaking, the

![Fig. 3. The fitting results of a representative lens in the spectral range from 632 to 633.5 nm reported from the offline calibration experiments. (a) The fitting results of the Mueller matrix based on the three theoretical models, (b) the six retrieval parameters and the fitting error. In Fig. 3(a), ‘measured’ corresponds to the measured Mueller matrix of a lens, ‘calc-1’ represents the proposed parametric model, ‘calc-2’ symbols an equivalent retarder with a rotatory angle [38], and ‘calc-3’ means the standard Lu-Chipman combination of a depolarizer, a retarder, and a diattenuator [30]. In Fig. 3(b), the variable ‘MSE’ means the fitting error in the spectral of 632–633.5 nm.](image-url)
linear phase retardance $\delta_1$ varies strikingly in the range from $0.00^\circ$ to $0.06^\circ$, and the circular retardance $\delta_2$ varies drastically in the range from $1.17^\circ$ to $4.67^\circ$. While the linear diattenuation $\varphi_1$ slowly varies in the range from $44.63^\circ$ to $44.67^\circ$, the circular diattenuation $\varphi_2$ smoothly varies in the range from $44.99^\circ$ to $45.01^\circ$, the depolarization-related coefficient $e$ gradually varies in the range from $0.998$ to $1.002$, and the depolarization-related coefficient $p$ varies in the range from $0.0$ to $1.2 \times 10^{-3}$. These results indicate that the magnitudes of relative changes in the parameters $\delta_1$ and $\delta_2$ are incredibly significant, while the magnitudes of relative changes in the parameters $\varphi_1$, $\varphi_2$, $e$, and $p$ are extremely small. Besides, both the linear diattenuation $\varphi_1$ and the circular diattenuation $\varphi_2$ are close to $45^\circ$, which indicates that both are weak. Considering that the deviation between $\varphi_1$ and $45^\circ$ is about 35 times larger than the deviation between $\varphi_2$ and $45^\circ$, a small linear diattenuation and a slight circular diattenuation can be confirmed.

The measured Mueller matrix in the spectral range from 400 to 800 nm was further fitted using the proposed parametric model. As shown in Fig. 4(a), the goodness-of-fit for the upper half of elements in the Mueller matrix is acceptable, which indicates that the proposed model can be applied to the offline calibration of the focusing lens in the visible light spectral range. Fig. 4(b) shows three polarization effect parameters $\delta_1$, $\delta_2$, and $p$, which possess obvious oscillatory features in the spectral dimension. In the spectral range of 400–800 nm, the linear phase retardance $\delta_1$ oscillates in the range from $-0.009$ to $0.018$, the circular retardance $\delta_2$ oscillates in the range from $-9^\circ$ to $15^\circ$, and the depolarization-related coefficient $p$ oscillates in the range from $-0.002$ to $0.004$. The significant oscillation in the parameters $\delta_1$, $\delta_2$, and $p$ is about 35 times larger than the deviation between $\varphi_2$ and $45^\circ$, a small linear diattenuation and a slight circular diattenuation can be confirmed.

Correspondingly, the fitting error of the Mueller matrix in the visible light spectral is shown in the bottom subfigure in Fig. 4(b), where the orange curve represents the magnitude of the fitting error. While, the fitting errors of the Mueller matrix obtained by using other three reference parametric model in the offline calibration are also shown in the subfigure, which are labeled by the open square marker, the open circle marker, and the open triangle marker, respectively. The first reference model is an orderly combination of a depolarizer, a circular retarder with a linear diattenuation, and an optical rotator. The second one is an orderly combination of a depolarizer, a circular retarder with a linear diattenuation, and an optical rotator. While the third one is an orderly combination of a depolarizer, a circular retarder with a linear diattenuation, and an optical rotator. While the fourth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the fifth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the sixth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the seventh one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the eighth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the ninth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator. While the tenth one is an orderly combination of a depolarizer, a linear retarder with a linear diattenuation, and an optical rotator.
The representative results of the first seven lenses.

Table 1

<table>
<thead>
<tr>
<th>Lens</th>
<th>Lens 1</th>
<th>Lens 2</th>
<th>Lens 3</th>
<th>Lens 4</th>
<th>Lens 5</th>
<th>Lens 6</th>
<th>Lens 7</th>
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<td>( \delta_1 ) (°)</td>
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<td>1.3850</td>
<td>-1.0970</td>
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<tr>
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<td>44.65</td>
<td>44.63</td>
<td>44.63</td>
<td>44.64</td>
<td>44.62</td>
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<tr>
<td>( \varphi_2 ) (°)</td>
<td>44.99</td>
<td>45.00</td>
<td>44.99</td>
<td>45.00</td>
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<tr>
<td>( \theta ) (°)</td>
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<td>-0.1721</td>
<td>-0.0535</td>
<td>0.2138</td>
<td>-0.7121</td>
<td>0.5408</td>
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<tr>
<td>( \epsilon )</td>
<td>0.9995</td>
<td>0.9993</td>
<td>0.9995</td>
<td>0.9994</td>
<td>0.9991</td>
<td>0.9994</td>
<td>0.9992</td>
</tr>
<tr>
<td>( p )</td>
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<td>-0.00338</td>
<td>-0.00139</td>
<td>0.00063</td>
<td>0.000898</td>
<td>-0.00079</td>
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<tr>
<td>( \text{MSE}^2 )</td>
<td>( 4.5 \times 10^{-6} )</td>
<td>( 3 \times 10^{-6} )</td>
<td>( 5.48 \times 10^{-6} )</td>
<td>( 2.9 \times 10^{-6} )</td>
<td>( 1.54 \times 10^{-6} )</td>
<td>( 4.36 \times 10^{-6} )</td>
<td>( 3.14 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

\( \delta_1 \) is an artificially introduced optical rotatory angle. \( \text{MSE}^2 \) represents the fitting error.

The polarization parameters of 29 lens pairs. (a) The linear phase retardance \( \delta_1 \) and the circular phase retardance \( \delta_2 \), (b) the linear diattenuation \( \varphi_1 \) and the circular diattenuation \( \varphi_2 \), (c) the artificially introduced optical rotatory angle \( \theta \), (d) the depolarization-related coefficients \( \epsilon \) and \( p \).

Fig. 5. The polarization parameters of 29 lens pairs. (a) The linear phase retardance \( \delta_1 \) and the circular phase retardance \( \delta_2 \), (b) the linear diattenuation \( \varphi_1 \) and the circular diattenuation \( \varphi_2 \), (c) the artificially introduced optical rotatory angle \( \theta \), (d) the depolarization-related coefficients \( \epsilon \) and \( p \).

0.0198, 45.0045 ± 0.0099, 0.9999 ± 0.0011, and 0.00005 ± 0.0010, respectively. The standard deviations of the parameters \( \delta_1 \), \( \delta_2 \), and \( \theta \) are at least 15 times the corresponding mean value, which indicate the highly dispersed distributions of these three polarization parameters. While the standard deviations of the parameters \( \varphi_1 \), \( \varphi_2 \), and \( \epsilon \) are at most one-thousandth of the corresponding mean value, which reflect the exceptionally concentrated distributions of these three parameters. Although the standard deviation of the polarization parameter \( p \) is at least 20 times its mean value, its variation within the distribution interval cannot enable the significant change of the Mueller matrix of the lens due to its relatively small mean value. Obviously, the results shown in both Fig. 5 and Table 1 exhibit an approximate two-fold relationship between the circular phase retardance \( \delta_2 \) and the introduced optical rotatory angle \( \theta \). Through evaluating the Pearson correlation coefficient matrix of the two parameters \( \delta_2 \) and \( \theta \) [40], the correlation coefficient can reach 0.9997, which reveals an extremely strong correlation. Meanwhile, the Pearson correlation coefficient of the two parameters in the spectral dimension can reaches 0.9999, which also reveals the strong correlation between them. The results validate the relationship \( \theta = \delta_2 / 2 \) in the proposed model. Thus, only the parameters \( \delta_1 \) and \( \delta_2 \) need to be considered as the retrieved quantities in the in-situ calibration, while the parameters \( \varphi_1 \), \( \varphi_2 \), \( \epsilon \), and \( p \) can be set as their average values reported by the above offline calibration. The procedure will be beneficial in suppressing the overfitting phenomenon in the in-situ calibration experiment.

Besides, although the polarization effect parameters of each lens are different, the differences in their Mueller matrix are mostly less than 0.001, which proves the rationality of the assuming that each lens is approximately the same.

4.2. In-situ calibration of the lens pair

As shown in Fig. 2(e), the marked area has been measured successively by the self-built M-MME in the large-spot measurement mode and the micro-spot measurement mode, and the measured light intensity is shown in Fig. 6. Fig. 6(a)–(d) exhibit the measured results of the SiO \(_2\) films with nominal thicknesses of 15 nm, 25 nm, 30 nm and 53 nm, respectively. The results shown in Fig. 6 indicate that the light intensity reported by the two measurement modes are significantly different, which directly confirms that the polarization effects of the lens pair can severely disturb the measurement results reported by the M-MME.

In the in-situ calibration experiment, only the linear phase retardance \( \delta_1 \) and the circular retardance \( \delta_2 \) are the fitting parameters, while the linear diattenuation \( \varphi_1 \), the circular diattenuation \( \varphi_2 \), the depolarization-related coefficients \( \epsilon \) and \( p \) are fixed as 44.6393°, 45.0045°, 0.9999 and 0.00005, respectively. Since the sample stage in the self-built instrument is not an ideal horizontal plane and the existing installation error, it is impossible to easily achieve a perfect focal plane through fine-tuning. Correspondingly, under the oblique incidence condition, it is difficult for the working area of the focusing lens surface to be consistent with that of the collection lens surface. As for the offline calibration experiment, the lens holding apparatus is carefully
customized, which allowing the precisely alignment and location of the focusing lens and the collection lens. These considerations will ensure the extremely small installation error, which further allows the ignoring of the actual deviation between the actual Mueller matrices of the two lens. However, such an approximation is no longer reasonable in the in-situ calibration experiment. Thus, the polarization effects of the collection lens will varies with the samples, which is consistent with the intrinsic characteristics of the polarization aberration of the lens [27]. In the actual in-situ calibration experiment, the polarization parameters $\delta_1$ and $\delta_2$ of the focusing lens has been set as the fixed values reported from the pre-fitting, while the polarization parameters $\delta_1$ and $\delta_2$ of the collimated lens and the thickness of the sample are considered as the retrieving parameters in the light intensity fitting process. The average values of the in-situ calibration results for the five measurement points ‘A’, ‘B’, ‘C’, ‘D’ and ‘E’ on different samples are shown in Table 2. These results again indicate that the light reflected from the surface of SiO$_2$ samples with different thicknesses may pass through different regions of the collection lens, so that the corresponding polarization effects exhibited by the collection lens are different among each other.

The measured thicknesses of different SiO$_2$ films reported by the above in-situ calibration process have been carefully compared with the results reported by the conventional in-situ calibration method and the results measured by the commercial MME. In the conventional in-situ calibration method, the polarization effects of the lens is optical equivalent to a retarder. Correspondingly, the comparison results are shown in Table 3. From the thicknesses $d_1$ and $d_2$ shown in Table 3, it can be easily found that there exists very high consistency in each thickness reported by the conventional or the proposed in-situ calibration method, which reveals the excellent repeatability of the self-built M-MME. However, it should also be noted that there is a relative deviation of at least 8% between the thicknesses reported by the conventional in-situ calibration method and those measured by the commercial MME. While the in-situ calibration method proposed in this article can reduce the relative deviation of thicknesses to 4.2% or less. Moreover, after the polarization effects of the lens pair is calibrated by using the proposed method, the relative deviation between the thicknesses reported by the conventional SiO$_2$ film and those measured by the commercial MME can be reduced to within 1.6%. Besides, it can also be noticed that with the increase of the thickness of the SiO$_2$ film, the relative deviation between the measurement results reported by the conventional in-situ calibration method between those measured by the commercial MME is increasingly larger, while the phenomenon does not appear in the comparison between the thickness reported by the proposed in-situ calibration method and the results measured by the commercial MME. The above analysis fully reveals the necessity of the in-situ calibration of the polarization effects of the lens pair, and also validates the effectiveness and feasibility of the proposed calibration method.

**Table 2**

<table>
<thead>
<tr>
<th>Nominal thickness (nm)</th>
<th>Lens L1</th>
<th>Lens L2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta_1$ (°)</td>
<td>$\delta_2$ (°)</td>
</tr>
<tr>
<td>15</td>
<td>3.3985</td>
<td>8.0117</td>
</tr>
<tr>
<td>25</td>
<td>3.3985</td>
<td>8.0117</td>
</tr>
<tr>
<td>30</td>
<td>3.3985</td>
<td>8.0117</td>
</tr>
<tr>
<td>53</td>
<td>3.3985</td>
<td>8.0117</td>
</tr>
<tr>
<td>57</td>
<td>3.3985</td>
<td>8.0117</td>
</tr>
</tbody>
</table>
In this work, a phenomenological parametric model based on the Mueller matrix decomposition has been described to propose the polarization effects of the lens pair in the M-MME. In the model, the single lens in the lens pair can be optically equivalent to a cascade system consisting of a circular retarder with slight diattenuation, a linear retarder with small diattenuation, a rotator, and a depolarizer. Correspondingly, by using six parameters including the circular phase retardance, the circular diattenuation angle, the linear phase retardance, the linear diattenuation angle, and two depolarization factors as the retrieving parameters, the polarization effects of the single lens can be fully modeled. Meanwhile, a comprehensive calibration method combining an initial offline based on a commercial spectral MME and a subsequent in-situ calibration based on self-built M-MME has been proposed to achieve the ingenious correction of polarization effects for the lens pair. Then, in order to demonstrate the effectiveness and feasibility of the proposed method, a series of thickness measurement experiments on SiO2 films have been carried out, in which the results indicate that the proposed calibration method could improve the relative deviation between the thickness measured by the self-built M-MME and those measured by the commercial MME to within 4.2%. Moreover, after the in-situ calibration of the polarization effects of the lens pair based on the proposed method, the relative deviation between the thickness of SiO2 film with a thickness larger than 25 nm measured by the self-built M-MME and those measured by the commercial MME is within 1.6%. Besides, the offline calibration results in the spectral range from 400 to 800 nm validate the extendibility of the proposed model in the broad spectrum. Predictably, the method can also be applied to the in-situ calibration of the focusing lens and high numerical aperture objective lens in the broadband Mueller matrix ellipsometer and the imaging Mueller matrix ellipsometer.

CRediT authorship contribution statement

Jiamin Liu: Investigation, Data curation, Formal analysis, Methodology, Validation, Writing – original draft. Zhou Jiang: Investigation, Data curation, Formal analysis, Methodology, Validation, Writing – original draft. Song Zhang: Data curation, Writing – review & editing. Tao Huang: Data curation, Formal analysis, Resources, Writing – review & editing. Hao Jiang: Conceptualization, Funding acquisition, Project administration, Supervision, Writing – review & editing. Shiyuan Liu: Writing – review & editing, Funding acquisition, Project administration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Table 3

<table>
<thead>
<tr>
<th>Nominal thickness (nm)</th>
<th>Commercial MME</th>
<th>Conventional in-situ calibration of M-MME</th>
<th>The proposed in-situ calibration of M-MME</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$d$ (nm)</td>
<td>$d_1$ (nm)</td>
<td>$\Delta d_1$ (nm)</td>
</tr>
<tr>
<td>15</td>
<td>14.97</td>
<td>13.79 ± 0.0067</td>
<td>1.18</td>
</tr>
<tr>
<td>25</td>
<td>25.38</td>
<td>23.39 ± 0.0068</td>
<td>1.99</td>
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<tr>
<td>30</td>
<td>30.33</td>
<td>27.28 ± 0.0026</td>
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<tr>
<td>53</td>
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<td>45.23 ± 0.0064</td>
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<tr>
<td>57</td>
<td>57.04</td>
<td>48.33 ± 0.0056</td>
<td>8.71</td>
</tr>
</tbody>
</table>

1 $\Delta d_1 = d - d_1$. 2 Dev$_1 = \Delta d_1 / d$. 3 $\Delta d_2 = d - d_2$. 4 Dev$_2 = \Delta d_2 / d$.

5. Conclusion

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References


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