A novel, to the best of our knowledge, optical method using a high-speed polarimetry is proposed for real-time attitude tracking in an ultra-large measurement range. The attitude metrology utilizes the field-of-view effect in birefringent crystals, which is known as the birefringence deviates with the field-of-view angle of polarized light. The basic principle of the metrology is presented via theoretical derivation and has been verified in the static retardance measurement experiments. With a resolution test, a temporal resolution of 0.4 ms per attitude measurement and an angular resolution up to 0.0025° are achieved. With the help of a bubble level, the attitude angles of an object attached with a birefringent wave plate are obtained in the dynamic experiments, which have achieved an accuracy better than 0.02°. Additionally, the angular velocity and acceleration of the real-time measured roll angle can be extracted simultaneously. The experimental results demonstrate that the proposed metrology has great potential and advantages in the real-time attitude sensing. © 2020 Optical Society of America

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The attitude angles are important parameters describing the motion of an object. In the fields of precision manufacturing [1], robotics control [2], and navigation of the aircraft [3], accurate and real-time monitoring of the attitude angles (yaw $\phi$, pitch $\theta$, and roll $\psi$) is very important. Traditional attitude metrologies are based on the combination of the attitude sensor information [4], such as gyroscopes and accelerometers. Since gyroscopes suffer from integral error [5], and accelerometers are susceptible to vibration [6], the systems are usually equipped with multiple sensors in order to accurately measure the attitude angle, which makes the systems complicated and costly.

Today, due to the advantages of being noncontact, flexible, low in cost, and highly precise and sensitive, the polarimetry methods show great potential in attitude metrology. Li et al. [7] and Gillmer et al. [8] presented a compact sensor for a roll angle, which achieved a $\pm 30^\circ$ and $43^\circ$ working range, respectively. MacPherson et al. presented a multicore fiber incorporating fiber Bragg grating strain sensors in each core as a fiber optic pitch and roll sensor [9], which achieved the resolutions of $\pm 2^\circ$ in roll $\phi$ and $\pm 15^\circ$ in pitch $\theta$. Saito et al. proposed a laser autocollimation method for measuring the attitude angle by using different diffraction light spots reflected from the diffraction gratings [10]. Although the autocollimator has a resolution of 0.01 arcsec and an accuracy of 0.5 arcsec, the measuring range is only 60 arcsec. In conclusion, the methods mentioned above have a limited measurement range and cannot achieve real-time attitude tracking. It is of great importance to develop an optical method to accurately measure the attitude angles in real time over an ultra-large measurement range.

In this Letter, we present a novel method for attitude tracking based on the high-speed polarimetry, which utilizes different phase modulations produced during the polarized light incident into the birefringent crystals along with different field-of-view angles. With the proposed method, the attitude angles can be obtained in an ultra-large measurement range, i.e., the roll angle of $0^\circ$–$360^\circ$, the theoretical ranges of the yaw and pitch are $-90^\circ$–$90^\circ$, respectively. The proposed method has been verified by the consistency achieved between the static measurements and the simulation. Additionally, in the dynamic measurement experiments, the attitude angles can be accurately measured, and the angular velocity and angular acceleration of the roll angle can be achieved simultaneously. The high precision and real-time measurement results have demonstrated the practicality and advantages of the proposed attitude metrology.

The experimental setup for the proposed attitude metrology consists of three parts: a high-speed Stokes polarimeter, a polarization state generator (PSG) and a sample stage, as described in our previous works [11]. The Stokes polarimeter with six parallel detection channels is based on the principle of spatial division of amplitude and can detect the Stokes vector of the probing light in several nanoseconds. The light source in the PSG is a 5 mW Red (632.8 nm) He–Ne Laser (THORLABS). The PSG can generate polarized light of any polarization state. The sample stage consists of a pitch adjustment stage, a yaw
adjustment stage, and a stepping motor, which can continuously change the \( \phi \) within the range of 0–360\(^\circ \), and fix the \( \theta \) and \( \varphi \) at an arbitrary value within the range of −90\(^\circ \)–90\(^\circ \) and −7\(^\circ \)–7\(^\circ \), respectively.

The birefringent device for the attitude metrology is a quartz compound zero-order wave plate, which consists of two multi-order wave plates whose optical axes are orthogonally aligned, as schematically shown in Fig. 1. In the birefringent wave plates, there are two refractive indices, i.e., the extraordinary index \( n_e \) and the ordinary index \( n_o \), which are in the directions parallel with and perpendicular to the optical axis, respectively. It is well known that the \( n_e \) will change with the incident angle of the light incident into the crystal, while the \( n_o \) remains unchanged [12]. If the optical axis of the \( i \)th element is parallel with the \( y_w - z_w \) axis, the effective refractive indices along \( y_w - z_w \) axis [12–14] can be expressed as

\[
\begin{align*}
\begin{cases}
n_{yi} = n_{zi} \left[ 1 + \left( \frac{1}{n_{zi}^2} - \frac{1}{n_{zi}^2} \right) \sin^2 \beta \cos^2 \phi \right]^{1/2}, \\
n_{zi} = n_{0i}.
\end{cases}
\end{align*}
\]

(1)

According to the propagation of the light in the wave plate shown in Fig. 1, the optical path difference can be written as

\[
L = n_{z1} |OA| - n_{z1} |OB| + n_{z2} |AC| - n_{z2} |BD| - |CD| \sin \beta
\]

\[= \sum_{i=1}^{2} d_i \left( \sqrt{n_{zi}^2 \sin^2 \beta - \sqrt{n_{zi}^2 \sin^2 \beta}} \right),
\]

(2)

where \( d_i \) is the thickness of the \( i \)th birefringent crystal, and \( n_{zi} \) and \( n_{zi} \) are the effective refractive indices of the \( i \)th birefringent crystal along the \( y_w \) and \( z_w \) axis directions, respectively. Additionally, it should be noted that the \( \theta \) and \( \varphi \) are coupled in the measurement of the incident angle \( \beta \), where \( \cos \beta = \cos \theta \cos \varphi \). Then, to decouple them, the \( \theta \) is measured by a bubble level, which has an accuracy of about 0.003\(^\circ \).

Finally, the incident light experiences a phase retardation change when traveling through the wave plate, as expressed:

\[
\delta(\beta, \phi) = \frac{2\pi}{\lambda} L = \frac{2\pi}{\lambda} \sum_{i=1}^{2} d_i \left( \sqrt{n_{zi}^2 \sin^2 \beta - \sqrt{n_{zi}^2 \sin^2 \beta}} \right) - \sqrt{n_{zi}^2 \sin^2 \beta}
\]

\[= \frac{2\pi}{\lambda} \cdot d_1 \left( \sqrt{n_{zi}^2 - \frac{n_{zi}^2 \cos^2 \phi + n_{zi}^2 \sin^2 \phi}{n_{zi}^2 \sin^2 \beta}} - \sqrt{n_{zi}^2 \sin^2 \beta} \right)
\]

\[= \frac{2\pi}{\lambda} \cdot d_2 \left( \sqrt{n_{zi}^2 - \frac{n_{zi}^2 \cos^2 \phi + n_{zi}^2 \sin^2 \phi}{n_{zi}^2 \sin^2 \beta}} - \sqrt{n_{zi}^2 \sin^2 \beta} \right).
\]

(3)

Inspired by the above principle, we can construct an analytical mapping between the attitude of a birefringent wave plate and its retardance. Namely, we can achieve attitude monitoring by using the field-of-view effect of birefringence. A practical birefringent device exhibits not only linear birefringence (LB), but also small linear diattenuation (LD) and circular birefringence (CB), when the light is obliquely incident [15,16]. To measure the attitude angles with polarimetry, the characterization of the wave plates in the Mueller matrix formalism can be expressed as [17]

\[
M_{wp}(\phi, \delta, \gamma) = M_{LB}(\phi, \delta) \cdot M_{CB}(\gamma) \cdot M_{LD}(\phi, \psi),
\]

where \( M_{LD} \) and \( M_{LB} \) are the Mueller matrix of a linear diattenuator and linear retarder, respectively, \( \psi \) and \( \delta \) are the diattenuation angle and linear retardance, respectively, and \( \phi \) is the azimuth of the fast axis. \( M_{CB} \) is the Mueller matrix of an optical rotator with the rotation angle \( \gamma \) [16,17]. It should be noted that since the wave plate used is supplied with antireflection coating, the LD caused by the interface diattenuation is negligible in the retardance measurement.

In principle, the light intensity matrix \( B \) measured by six photomultiplier tubes can be obtained by multiplying the Stokes vector \( S_{in} \) of the incident light, the Mueller matrix of sample \( M \), and the demodulation matrix \( A \) of the FPMS [18]. In order to avoid the errors caused by CB and simplify the data analysis process, the \( \delta \) of the wave plate is extracted from Stokes parameters \( S_3 \), when the polarization state of the incident light is left-hand circular polarization. Then the \( \phi \) can be extracted from \( S_1/\sin(\delta) \) or \( S_3/\sin(\delta) \), as given by

\[
S_{out} = M_{WP} \cdot [1 0 0 1]^T = (A^T A)^{-1} A^T B
\]

\[= [1 \sin(2\phi) \sin(\delta) \cos(2\phi) \sin(\delta) \cos(\delta)]^T.
\]

(5)
Based on the extracted results of $\delta$ and $\phi$, we can obtain the $\beta$ with Eq. (3). Since the $\theta$ is measured by a bubble level, the $\varphi$ can be calculated by

$$\varphi(t) = \arccos \left[ \frac{\cos \beta(t)}{\cos \theta(t)} \right]. \quad (6)$$

The proposed attitude metrology is theoretically feasible with the $\phi$ in the range of $0^\circ$–$360^\circ$, and $\theta$ and $\varphi$ in the range of $-90^\circ$–$90^\circ$. However, limited by the clear aperture (CA) of the wave plate and the travel of the adjustment stages, we can only demonstrate our method with $\theta$ in the range of $-7^\circ$ and $7^\circ$, and $\varphi$ in the range of $-40^\circ$–$40^\circ$ in our experiments.

The system is carefully in situ calibrated before performing the measurement experiments [11]. Most of the systematic errors and random noise have been compensated for in the calibration process. Then the largest error source remaining in the attitude metrology is the incident imperfect circularly polarized light. To get a circularly polarized light, we first maintain a relative angle of about $45^\circ$ between the optical axis of a polarizer and that of a quarter-wave plate without samples on the polarimeter. Due to the deviation of the relative angle $\Delta \theta_{ci}$ and the deviation of the retardance $\Delta \delta_{ci}$ of the wave plate, the output light is non-ideal circularly polarized light. Then, to compensate for the deviations, the polarizer and wave plate are rotated simultaneously in the same increments. Meanwhile, the function Eq. (7) is adopted to estimate the difference between the measured and calculated Stokes vectors. By minimizing the function, we can obtain the $\Delta \theta_{ci}$ and $\Delta \delta_{ci}$ in the fitting procedure. Finally, a relatively ideal circularly polarized light can be output by compensating for the deviations:

$$\chi^2 = \sum_{q=1}^{N_m} \sum_{p=0}^{3} \left[ \frac{S_{q,p}^m - S_{q,p}^c (\Delta \theta_{ci}, \Delta \delta_{ci})}{\sigma(S_{q,p})} \right]^2, \quad (7)$$

where $N_m$ is the total number of the increment measurements, the superscripts “$m$” and “$c$” represent the measured and calculated results, respectively, the subscript “$q$, $p$” represents the $p$th Stokes parameter in the $q$th increment measurement, and $\sigma(S_{q,p})$ is the standard deviation of the measured Stokes vector.

Fig. 2. Retardance of the wave plate under different attitude angles: subscripts “$m$” represent the measured results; subscripts “$Er$” represent retardance measurement errors, which are defined as simulated results minus the measurement results. (i) $\psi$ is in the range of $-34^\circ$–$39^\circ$ when the $\theta$ is $0^\circ$ and the $\phi$ is $100^\circ$; (ii) $\phi$ is in the range of $-90^\circ$–$90^\circ$ when the $\theta$ is $0^\circ$ and the $\psi$ is $20^\circ$; (iii) $\theta$ is in the range of $\pm 7^\circ$ when the $\psi$ is $0^\circ$ and the $\phi$ is $20^\circ$.

To verify the correctness of the proposed attitude metrology, the retardance of the wave plate at different attitude angles (i.e., field-of-view angles) is measured with the Stokes polarimeter, and the results are compared with theoretical calculations. Specifically, we make one of the attitude angles vary within a certain range in the experiments, while the other two remain unchanged. It can be observed from Fig. 2 that the measured results are consistent with the simulation results, and the measured error of the retardance is less than $2^\circ$. Besides, the retardance changes versus $\psi$ and $\theta$ have the same waveform. This is because in the retardance calculation formula [Eq. (3)] the changes of $\psi$ and $\theta$ induce the same variation in the incident angle. It is impossible to distinguish them by the measured $\phi$ and the retardance alone, which is the reason for measuring the $\theta$ with the bubble level. Moreover, the smaller the incident angle is, the smaller is error in the retardance measurement.

The relatively large error in retardance measurement at a large incident angle results from the incidence of the edge region of the wave plate limited by the effective CA of the wave plate and size of the mount. A larger measurement range can be achieved by increasing the CA of the wave plate and the size of the mount.

We also perform an angular resolution test of the $\phi$ with the instruments. As an example, the angular velocity of the motor is kept at $20^\circ$/s, the sampling frequency of the polarimeter is 2.5 KHz, and the measurement of the $\phi$ is performed with $\psi = 0^\circ$ and $\theta = 0^\circ$. Additionally, the results are shown in Fig. 3. It can be observed that the measured results are consistent with the preset results. However, there is a relatively large error of about $0.02^\circ$. Since the measurement of the polarimeter is a direct measurement of the roll angle at a single time point, the error is not the cumulative error, and it is more likely due to the unstable rotation of the motor. Meanwhile, the relative errors fluctuate between $\pm 0.02^\circ$, and the error will cancel out each other when the stepper motor runs for a long time. Besides, the measured resolution of the $\phi$ is better than $0.0025^\circ$, which fluctuates around the preset value $0.0024^\circ$. Additionally, the increment measurement accuracy of the $\phi$ is better than $0.0001^\circ$. Additionally, further improvements in motor control system will result in more accurate measurement results.

Finally, the practicality of the proposed attitude metrology is demonstrated through a varying-velocity rotation real-time measurement experiment under arbitrary attitude angles. As an example, the $\phi$ driven by the motor changes with time, the $\theta$ which is determined by a bubble level is fixed at $4^\circ$, and the $\psi$ is fixed at $6^\circ$. Meanwhile, the sampling frequency of the polarimeter is 2.5 KHz.

Fig. 4 presents the measurement results on the $\phi$ changing with the time in the dynamic experiments. It can be observed that the $\phi$ increases with the time at different rates, as shown Fig. 4(a). In order to compare the measured results with the
over time, (b) $\alpha_\phi$ over time, (c) errors of $\omega_\phi$, and (d) $\alpha_\phi$ over time.

Fig. 4. Results about the $\phi$ in the dynamic experiments: (a) the $\phi$ over time, (b) $a_\phi$ over time, (c) errors of $\omega_\phi$, and (d) $\alpha_\phi$ over time.

dynamic experiments. The proposed method has been proven to be feasible with the $\phi$ in the range of $0^\circ$–$360^\circ$. Although we can only show the measurement range $-7^\circ$–$7^\circ$ for $\theta$ and $-40^\circ$–$40^\circ$ for $\phi$ due to the limit of the experimental setup, their theoretical detectable ranges approach $-90^\circ$–$90^\circ$. The results show that all the attitude angles of the object attached with a birefringent crystal can be obtained with an accuracy of $0.02^\circ$. As an additional benefit, the angular velocity and acceleration of the real-time measured roll $\phi$ can be extracted simultaneously, which is expected to pave the way for real-time attitude sensing.

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