Proof of principle of an optical Stokes absolute roll-angle sensor with ultra-large measuring range

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A B S T R A C T

We present a novel optical roll-angle sensor called Stokes roll-angle sensor. The measurement principle relies on modulating the roll angular displacement into the change of state of polarization (SoP) of polarized probe light by a quarter-wave plate (QWP). The SoP of the emerging elliptically polarized light from QWP is then detected by a Stokes polarimeter. Based on the detected Stokes vector, the roll angle can be extracted. Experimental results have shown that the proposed Stokes roll-angle sensor can realize absolute angle measurement in an unprecedented range of 180° with 0.02° resolution. Further expanding the measuring range to 360° is realizable by marking a marker on the QWP to distinguish its two half parts divided by the fast-axis in the rotation. Other benefits of the proposed Stokes roll-angle sensor include simplicity, compactness, high speed, and low cost.

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1. Introduction

Angle is an important quantity for description of geometries and motions of objects. Precise angle measurement is often required in engineering and scientific instruments, such as machine tools and coordinate measuring machines [1], navigation of robots [2–5], autonomous docking of two spacecraft [6], and torsion balance apparatus for laser interferometer gravitation-wave observatory [7]. Angle measurement can be carried out in an incremental manner [8] or in an absolute manner [5,9]. Compared with the incremental manner, absolute angle measurement gives the exact angular position of an object and is becoming increasingly important in many applications such as large-scale precision manufacturing systems and multi-articulated robots. The rotary encoder is commonly employed for absolute angle measurement over a full range of 360° with a resolution depending on the graduation spacing of the disk [10,11]. Nevertheless, it should be noted that the manufacturing of an accurate disk for an absolute rotary encoder is usually quite complicated process. Among the various available methods, optical methods have currently become a major method for angle measurement due to the advantages of high-sensitivity, flexibility of design, and without contact.

As one of the three types of angles (roll, yaw, pitch), roll angle is always deemed to be the most difficult angular displacement to measure using optical methods, since the roll angular displacement is perpendicular to the optical axis of the probe beam and is difficult to be sensed by the probe beam. Optical roll-angle measurement is commonly carried out by translating roll angular displacement into either a change of optical path difference of two interference beams (angular interferometers [12–15]), or a focal spot shift on a position-sensitive photodetector (laser autocollimators [16–19]), or a change of SoP of polarized probe light [20–23]. Among these methods, the polarization-based measurement can provide a high sensitivity in a relatively large measuring range. Li et al. presented a compact roll-angle sensor in a ±30° working range with 0.01° resolution [20], which used a Faraday rotator to modulate polarization direction of the probe light. Gillmer et al. further improved the design of the roll-angle sensor in [20] by replacing the Faraday rotator therein with an acousto-optic modulator and presented a roll-angle sensor in a working range of 43° with 0.002° resolution [21]. To the best of our knowledge, the roll-angle sensors presented in [20,21] represent the largest achievable measuring range in optical roll sensing in the reported literature.

In this work, we propose an optical Stokes roll-angle sensor, where the roll angular displacement is modulated into the change of SoP of polarized probe light by a QWP. The SoP of the emerging elliptically polarized light from the QWP is then detected by a complete Stokes polarimeter. Based on the detected Stokes vector, the roll angle can be finally extracted. It is shown that the proposed Stokes roll-angle sensor can realize absolute angle measurement.
in an unprecedented range of 180°. Moreover, the measuring range can be further expanded to 360° by making a marker on the QWP to distinguish its two half parts divided by the fast-axis in the rotation. The proposed angle-sensor also takes the advantages of simplicity, compactness, high speed, and low cost.

The remainder of this paper is organized as follows. Section 2 presents the measurement principle of the proposed Stokes roll-angle sensor. Section 3 presents experimental details, including the development of a prototype of the proposed Stokes roll-angle sensor, an in-situ regression method for calibration of the developed sensor, as well as experimental results to show the measuring range and resolution of the developed sensor. Some concluding remarks are finally drawn in Section 4.

2. Measurement Principle

Fig. 1 shows the scheme of the proposed Stokes roll-angle sensor, which consists of a light source, a linear polarizer (P), a quarter-wave plate (QWP), a complete Stokes polarimeter. The QWP is fixed with a rotating component under test. The rotating QWP modulates the incoming linearly polarized light into an elliptically polarized light with the SOP determined by the orientation angle θ of the fast-axis of the QWP. The orientation angle θ of the QWP is related to the roll angle θ of the rotating component under test by θ = θ + θ0, with θ0 being the initial orientation angle of the fast-axis of the QWP. The complete Stokes polarimeter is employed to detect the SOP of the elliptically polarized light emerging from the QWP. The word “complete” here is used to emphasize that the employed Stokes polarimeter should be able to collect the full Stokes vector \( \mathbf{S} = [S_0, S_1, S_2, S_3]^T \) of the elliptically polarized light, with “T” representing the matrix transpose. The roll angle θ can be finally obtained according to the measured Stokes vector.

According to the Stokes-Mueller formalism, the Stokes vector \( \mathbf{S} \) associated with the elliptically polarized light emerging from the QWP can be represented by

\[
\mathbf{S} = \mathbf{R}(\theta') \mathbf{M}_{\text{QWP}} \mathbf{R}(\theta) \mathbf{M}_{\text{P}} \mathbf{S}_0,
\]

where \( \mathbf{S}_0 \) denotes Stokes vector of the light source. Assuming that the light emitted from the light source is perfectly random (unpolarized), \( \mathbf{S}_0 = [I_0, 0, 0, 0]^T \) with \( I_0 \) being the intensity of the emitted light. \( \mathbf{M}_\text{P} \) and \( \mathbf{M}_{\text{QWP}} \) represent Mueller matrices of the polarizer and QWP, respectively, which are given by

\[
\mathbf{M}_\text{P} = \frac{1}{2}
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
\mathbf{M}_{\text{QWP}} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{bmatrix}.
\]

\( \mathbf{R}(\theta) \) is the Mueller rotation transformation matrix for rotation by an arbitrary angle \( \theta \) and is given by

\[
\mathbf{R}(\theta) =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos(2\theta) & \sin(2\theta) & 0 \\
0 & -\sin(2\theta) & \cos(2\theta) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

According to Eq. (1), when the transmission-axis orientation angle of the polarizer \( \theta_0 = 0^\circ \), we obtain

\[
S_1 = \left[1 + \cos(4\theta') \right]/2, \tag{4a}
\]

\[
S_2 = \sin(4\theta'/2), \tag{4b}
\]

\[
S_3 = \sin(2\theta''). \tag{4c}
\]

According to Eq. (4), \( \theta' \) can be obtained from any of the three Stokes-vector parameters \( S_1, S_2, \) and \( S_3 \). As an illustrative example, we obtain \( \theta' \) from \( S_1 \) and meanwhile use \( S_2 \) and \( S_3 \) to distinguish the intervals of \( \theta' \). Considering that \( \arccos(x) \) returns values in the interval \([0, \pi]\) for \( x \in [-1, 1] \), we then obtain \( \theta' \) in the interval \([0, \pi]\) by

\[
\theta' =
\begin{cases}
\frac{\pi}{2} - \frac{1}{4} \arccos(2S_1 - 1), & S_2 > 0, S_3 > 0 \\
\frac{\pi}{2} - \frac{1}{4} \arccos(2S_1 - 1), & S_2 < 0, S_3 < 0 \\
\pi - \frac{1}{4} \arccos(2S_1 - 1), & S_2 > 0, S_3 < 0 \\
\pi - \frac{1}{4} \arccos(2S_1 - 1), & S_2 < 0, S_3 > 0
\end{cases}.
\]

Knowing the initial orientation angle \( \theta_0 \) of the fast-axis of the QWP, which is determined after the installation and will stay constant in the rotation of the QWP, the roll angle \( \theta \) of the rotating component under test can be further obtained by \( \theta = \theta' - \theta_0 \). Evidently, here the measurement of the roll angle is an absolute measurement. To be intuitive, Fig. 2 presents the representation of Stokes vector \( \mathbf{S} \) of the elliptically polarized light emerging from the QWP varied with the roll angle \( \theta \) on the Poincaré sphere.

When the transmission-axis orientation angle of the polarizer \( \theta_0 \neq 0^\circ \), we can obtain

\[
S_1 = \left[\cos(2\theta_0) + \cos(2\theta_0 - 4\theta') \right]/2, \tag{6a}
\]

\[
S_2 = \left[\sin(2\theta_0) - \sin(2\theta_0 - 4\theta') \right]/2, \tag{6b}
\]

\[
S_3 = \sin(2\theta' - 2\theta_0). \tag{6c}
\]
It can be demonstrated that the judging criteria given in Eq. (5) are still applicable if $P_0$ is close to 0. To obtain the values of $\theta'$, one only needs to replace the term $\arccos(2S_1 - 1)/4$ in Eq. (5) with $[2P_0 + \arccos(2S_1 - \cos 2P_0)]/4$. It should be noted from Eq. (4) that when $\theta' = 0$ or $\theta' = \pi/2$, $S_2 = 0$ and $S_3 = 0$. Therefore, the judge criteria in Eq. (5) will be invalid in these two cases, which can also be observed from Fig. 2. To distinguish $\theta' = \pi/2$ from $\theta' = 0$, we can set $P_0$ to slightly deviate from 0 so that the judging criteria in Eq. (5) are applicable for most of the values of $\theta'$ except $\theta' = 0$ and $\theta' = \pi/2$. For the special cases of $\theta' = 0$ and $\theta' = \pi/2$, we know from Eq. (6) that $S_2 = 0$, $S_3 = -\sin(2P_0)$ in the case of $\theta' = 0$, and $S_2 = 0$, $S_3 = \sin(2P_0)$ in the case of $\theta' = \pi/2$. It indicates that we can distinguish the special cases between $\theta' = 0$ and $\theta' = \pi/2$ according to the value of $S_2$ and the sign of $S_3$. As an example, Fig. 2 presents the determination of $\theta' = 0$ and $\theta' = \pi/2$ when $P_0 = -2$.

The reason that the Stokes roll-angle sensor can only obtain the values of $\theta'$ in the interval $[0, \pi]$ while not in the complete interval $[0, 2\pi]$ is that the fast-axis of QWP after a 180° rotation coincides with that at the initial rotation position. If we could make a marker on QWP so as to distinguish its two half parts divided by the fast-axis in the rotation, it is probable to expand the measuring range from $[0, \pi]$ to $[0, 2\pi]$. It should be pointed out that the QWP in Fig. 1 (top) could not be replaced with a half-wave plate (HWP), otherwise, the measuring range will be reduced to $[0, \pi/2]$ since the last Stokes vector element $S_3$ is always 0 for a HWP. Fig. 2 also presents the representation of the Stokes vector $S$ of the polarized light emerging from a HWP varied with the roll angle $\theta$ on the Poincaré sphere. As can be observed, the trajectory of $S$ is a circle along the equator. In fact, other waveplates could also be adopted, provided that the trajectory of the Stokes vector $S$ of the elliptically polarized light emerging from the waveplate forms an “8” shape on the Poincaré sphere.

The Stokes polarimeter is employed to measure the Stokes vector $S$ associated with the elliptically polarized light emerging from the QWP. The measurement process can be generally represented as

$$ I = AS, $$

where $I$ denotes the vector of measured fluxes by the detector and $A$ is the instrument matrix determined by the polarizing components comprised of the polarimeter system. According to Eq. (7), the Stokes vector $S$ can be measured by

$$ S = A^{-1}I, $$

where $A^* = (A^*A)^{-1}A^*$ is the Moore-Penrose pseudo-inverse of $A$.

Since the Stokes vector has four elements, it needs to carry out at least four measurements for a complete Stokes polarimeter. There are many types of available Stokes polarimeters [24]. Fig. 1 (bottom) presents two schemes of the Stokes polarimeter. Fig. 1(a) presents the scheme of a rotating-compensator polarimeter, and Fig. 1(b) presents the scheme of a four-channel polarimeter. In Fig. 1(a), the detected intensity is a temporally periodic function, whose Fourier harmonic coefficients are directly related with the undetermined Stokes-vector elements. In Fig. 1(b), the four Stokes-vector elements are simultaneously determined from the four detector measurements. Compared with the four-channel polarimeter, the rotating-compensator polarimeter has a simple layout and a compact size. However, since the four Stokes-vector elements are determined simultaneously, the Stokes roll-angle sensor based on a four-channel polarimeter will be well-suited for high-dynamic metrology. Additionally, in practice, the measured Stokes-vector elements are usually normalized to the first element $S_0$, which denotes the total light intensity of the polarized light. The roll angles calculated by Eqs. (4)-(6) with the normalized Stokes-vector elements will be relatively immune to noise fluctuation.

3. Experiments

3.1. Experimental setup

To demonstrate the performance of the proposed Stokes roll-angle sensor, as shown in Fig. 3, we have developed a prototype of the Stokes roll-angle sensor based on a rotating-compensator polarimeter. The light with a wavelength of 633 nm from a high-stability light source (Eq-99XFC, Energetiq Technology, Inc., USA) was made to pass successively through the polarizer $P$ (PGT5012, Union Optics, Inc., China) and the QWP (WPQ05M-633, Thorlabs, Inc., USA). The QWP was mounted in a hollow-shaft servo-motor (HO-63-A-44-A-E-000, Applimotion, Inc., USA) with a resolution of $0.005^\circ$ to simulate the rotation of the rotating component under test. The emerging polarized light from the QWP was then made to pass successively through the rotating compensator $C_r$ (mounted in a same hollow-shaft servo-motor to the QWP), the analyzer $A$ (PGT5012, Union Optics, Inc., China), and finally to enter into the detector $D$ (PCO.edge 5.5, PCO, Inc., Germany). In practice, there will always be uncertainties associated with the measured fluxes $I$ and with the instrument matrix $A$ in Eq. (7). The uncertainties in $A$ arise from inaccurate calibration of system parameters of the Stokes polarimeter (See Section 3.2). To minimize the effect of small errors in $I$ and $A$ on the calculated Stokes vector $S$, the rotating compensator was an optimally-designed Quartz biaxial plate consisting of two zero-order quartz Quartz waveplates by minimizing the condition number of $A$, $\text{cond}(A) = \|A\| \|A^{-1}\|$, with $\|\|$ denoting the maximum norm of a matrix [25]. The reason that the 633 nm wavelength was chosen in the experimental setup was just to accommodate the employed QWP. Other wavelengths could also be chosen. In addition, although a camera was employed here as the detector, it should be noted that other detectors such as photomultiplier tubes could also be used. As shown in Fig. 3, currently the experimental setup seems to be pretty large, since it was not specially designed but was revamped from a previous instrument [23]. The size of the proposed Stokes roll-angle sensor rests with the employed Stokes polarimeter. For the rotating-compensator Stokes polarimeter, its size is mainly limited by the motor driving the rotating-compensator $C_r$ and the detector. The size of the
3.2. Sensor calibration

The system parameters that need to be calibrated mainly include the initial transmission-axis orientation angles of the polarizer $P_0$ and the analyzer $A_0$, the initial orientation angles of the fast axes of the QWP $\theta_0$ and the rotating compensator $C_0$, as well as the phase retardance of the rotating compensator $\delta$. The angles $P_0$, $A_0$, $\theta_0$ and $C_0$ are all defined relative to the horizontal plane, which are determined after installation of the corresponding components. If we take the QWP as the sample, the system that consists of a polarizer $P$, a rotating compensator $C$, and an analyzer $A$ can be regarded as a rotating-compensator ellipsometer (RCE) [26] in a straight-through configuration. The reported calibration methods for RCE can also be adopted to calibrate the Stokes roll-angle sensor [26–28]. Here, we propose an in-situ regression method to calibrate the system parameters $P_0$, $A_0$, $\theta_0$, $C_0$ and $\delta$ by fitting the measured Fourier coefficients ($\alpha_2$, $\beta_2$, $\alpha_4$, $\beta_4$) obtained from the modulated light intensity to the theoretically calculated Fourier coefficients. To improve calibration accuracy, the QWP is rotated in a range from its initial orientation $\theta_0$ with a certain increment in the calibration. The Fourier coefficients obtained from the measured modulated light intensities associated with all the rotated orientations of the QWP are as an ensemble fitted to the corresponding theoretically calculated Fourier coefficients. A $\chi^2$ function is adopted to estimate the fitting error between the measured and theoretically calculated Fourier coefficients in the fitting procedure, which is defined as

$$
\chi^2 = \sum_{m=1}^{K} \sum_{n=1}^{2} \left\{ \frac{\alpha_{m,2n}^{\text{exp}} - \alpha_{m,2n}^{\text{calc}}(P_0, A_0, \theta_0, C_0, \delta)}{\sigma(\alpha_{m,2n})} \right\}^2 
+ \left\{ \frac{\beta_{m,2n}^{\text{exp}} - \beta_{m,2n}^{\text{calc}}(P_0, A_0, \theta_0, C_0, \delta)}{\sigma(\beta_{m,2n})} \right\}^2,
$$

where $K$ denotes the total number of rotated orientations of QWP, $\alpha_{m,2n}^{\text{exp}}$ and $\beta_{m,2n}^{\text{exp}}$ are the Fourier coefficients obtained from the measured modulated light intensity associated with the $m$-th rotated orientation of QWP, $\alpha_{m,2n}^{\text{calc}}$ and $\beta_{m,2n}^{\text{calc}}$ are the corresponding theoretically calculated Fourier coefficients, and $\sigma(\alpha_{m,2n})$ and $\sigma(\beta_{m,2n})$ are the standard deviations of the measured Fourier coefficients. The Levenberg-Marquardt algorithm [29] was adopted for the solution of the regression problem described above.

4. Results and discussion

Fig. 4(a) presents the measured roll angles when the QWP rotates over 360° with an increment of 20°. The proposed Stokes roll-angle sensor was also tested at the input roll angles of 179° and 359° to demonstrate its measuring range; (b) Absolute measurement errors between the input and measured roll angles in Fig. 4(a). The inserted equations in Fig. 4(a) are the fitted linear equations and coefficients of determination ($R^2$).
angle measurement in an ultra-large measuring range of [0°, 180°]. Moreover, the limitation to the measuring range is just because the fast-axis of the QWP coincides with that at the initial rotation position. If we could make a marker on the QWP to distinguish its two half parts divided by the fast-axis in the rotation, it is possible in principle for us to further expand the measuring range to [0°, 360°]. Fig. 4(b) presents the measurement errors between the input and measured roll angles shown in Fig. 4(a). The measurement error here is defined as the absolute difference between the input and measured roll angles, which is mainly attributed to the calibration errors of system parameters. As described in Section 3.2, since the sensor calibration is based on the fitting between the measured and theoretically calculated Fourier coefficients, any errors in the measured Fourier coefficients will finally propagated into the calibrated system parameters in the fitting procedure. The errors in the measured Fourier coefficients include random noise in the measured light intensity and systematic errors induced in the installation. As can be observed from Fig. 4(b), the measurement errors are less than 0.35° over the whole measuring range. Further improvement of calibration accuracy would lead to further decrease of measurement errors.

We also performed a resolution test to the developed Stokes roll-angle sensor by controlling the servo-motor rotated with different incremental steps. Fig. 5(a) and (b) present the measured roll angles as a function of input roll angles with an increment of 0.03° and 0.02°, respectively. As can be observed from Fig. 5(a), the developed sensor can clearly distinguish the input roll angles with a step of 0.03°, which indicates that the developed sensor has a measurement resolution of better than 0.03°. For the input roll angles with a step of 0.02° shown in Fig. 5(b), the developed sensor can distinguish most of the input roll angles except two data points. It suggests that the measurement resolution of the developed sensor shall be about 0.02°. The limitation to the measurement resolution of the developed Stokes roll-angle sensor is mainly the shot noise of the light itself due to the relatively high output power of the employed light source (100 μW at 633 nm wavelength) and the relatively low readout noise of the employed camera (1.0 e- med.). Other techniques such as weak-value and weak-value-emulated amplification [31–33] may be adopted to further improve the measurement performance of the Stokes roll-angle sensor.

5. Concluding remarks

In this paper, we have proposed a novel optical roll-angle sensor, which we call the Stokes roll-angle sensor. The detailed measurement principle as well as an in-situ regression calibration method have been presented. Experimental results have shown that the proposed Stokes roll-angle sensor can realize absolute angle measurement in a range of 180° with 0.02° resolution, which is the largest measuring range in optical roll sensing to date. Moreover, the measuring range can be further expanded to 360° by making a marker on the angle-sensing component, a QWP, to distinguish its two half parts divided by the fast-axis in the rotation. Due to the exceptional advantages of the proposed Stokes roll-angle sensor, it is expected to gain wide applications. It should be noted that this paper is a report of the first step of the research where the proposal and preliminary verification of the proposed Stokes roll-angle sensor were primarily focused. Future work will be carried out to further improve the calibration accuracy and measurement resolution as well as to reduce the size of the sensor. In addition, the stability of the sensor calibration and measurement will also be investigated.

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