Struct Multidisc Optim (2011) 44:751–759
DOI 10.1007/s00158-011-0690-3

INDUSTRIAL APPLICATION

Simultaneous optimization of cast part and parting direction using level set method

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Received: 6 April 2011 / Revised: 2 June 2011 / Accepted: 1 July 2011 / Published online: 10 August 2011

Abstract Parting direction is one of the main parameters that significantly affect mouldability and manufacturing costs of a cast part. In conventional optimal design of cast part, a parting direction is pre-selected by a designer and fixed throughout the optimization. However, when the optimization is performed with a different parting direction, the resulting design will also be different, and more importantly it will end up with different working performance. Therefore, we take the parting direction as a design variable in the optimization of a cast part so that the working performance can be optimized as much as possible. With these goals, a level set based method is proposed for the simultaneous optimization of cast part and parting direction. In each iteration, an optimal parting direction is first computed for the current structure, then the boundary of the current structure is updated by a design velocity that guarantees the design be moldable with the optimal parting direction. Therefore, although the parting direction may be changed during the optimization, the structure will always be moldable in the current parting direction. Numerical examples are provided in 3D.

Keywords Cast part · Molding constraint · Parting direction · Structure optimization · Level set

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1 Introduction

In casting process, molten liquid is poured into and solidifies in a cavity enclosed by molds, and one obtains the cast part when the molds are removed. An important issue in the casting process is that a cast part should have a proper geometry so that the molds can be removed, or the so-called molding constraint can be satisfied. The molding constraint is an important manufacturing problem that requires careful consideration in the design of a cast part, since a structure should be designed in a way that it can be easily manufactured via intended techniques. In fact, considering manufacturability early in the design cycle reduces the costs of product development, shortens its development time, and improves its quality.

Optimization is an effective tool for obtaining high performance structures. During the past decades, size optimization, shape optimization, and topology optimization have been extensively developed. While the performance of a structure can be improved via conventional structure optimization, another aspect in the design of a structure—manufacturing—should also be carefully treated. In other word, the optimal design of a cast part should not only optimize the performance of the cast part but should also ensure the cast part have proper geometry that satisfies the molding constraint.

Much effort has been made for integrating the molding constraint of casting process into structure optimization. TopShape (Baumgartner et al. 1992; Mattheck 1990; Harzheim and Graf 1995), a program developed at the International Development Center of Adam Opel and based on CAO (Computer Aided Optimization) and SKO (Soft Kill Option) (Baumgartner et al. 1992; Mattheck 1990; Harzheim and Graf 1995), is the first program that successfully incorporated the molding constraint of casting process into structure optimization (Harzheim and Graf 2002, 2005, 2006). Several topology control algorithms (connectivity control, growth direction control, thickness control, et al.) were introduced in the TopShape. Based on the SIMP (Solid Isotropic Microstructure with Penalization) method (Bendsøe 1989; Rozvany et al. 1992; Bendsøe and Sigmund 2003), Zhou et al. proposed a mathematical formulation of the molding constraint that ensure the material densities in the lower positions along the parting direction to be bigger than those in the upper positions (Zhou et al. 2001). This approach is available in OptiStruct (Altair Engineering, Inc. 2002). Leiva et al. proposed a novel design parametrization that explicitly incorporates parting direction into its design variables for the topology optimization of cast part (Leiva et al. 2004a, b) and implemented it in GENESIS (Leiva et al. 1999). Based on the level set method, we proposed in our previous study a restricted motion of boundary of structure for the optimization of cast part (Xia et al. 2010).

The present paper extends the study of optimization of cast part by simultaneously optimizing the parting direction. In conventional optimal design of cast part, the parting direction is pre-selected by a designer and fixed throughout the optimization. With such approach, when the optimization is performed with a different parting direction, the resulting design will also be different, and more importantly it will end up with different working performance. In other word, the working performance depends on the parting direction. Therefore, it will be better if we take the parting direction as a design variable of optimization so that the working performance of a cast part can be optimized as much as possible.

The paper is organized as follows. In Section 2 the level set method is briefly described. In Section 3 the casting process is briefly described, and the manufacturing issue is analyzed. In Section 4 the problem of simultaneous optimization of cast part and parting direction is defined. In Section 5, the numerical implementation is introduced. Section 6 gives numerical examples and discussion. Section 7 concludes this paper.

2 The level set method

Level set method, first introduced by Osher and Sethian (1988), is a method for numerical simulation of motion of interfaces in two or three dimensions. The level set method is transparent to topological changes, which is significant for topology optimization. With such transparent treatment of topological change, the difficult topology optimization problem can be transformed to a relatively easier shape optimization problem. It has caught extensive attentions since the seminal papers (Sethian and Wiegmann 2000; Osher and Santosa 2001; Allaire et al. 2002, 2004; Wang et al. 2003).

Let $\Omega \subseteq \mathbb{R}^d$ (d =2, 3) be the region occupied by a structure. The boundary of the region is represented implicitly through a level set function $\Phi(x)$ as its zero isosurface or zero level set, i.e., $\{x \in \mathbb{R}^d \mid \Phi(x) = 0\}$ ($d = 2$ or 3), and $\Phi(x)$ can be used to define the inside and outside regions as follows

$$\begin{aligned}
\Phi(x) = 0 \quad &\Longleftrightarrow \quad \forall\, x \in \partial\Omega \cap D \\
\Phi(x) < 0 \quad &\Longleftrightarrow \quad \forall\, x \in \Omega \\
\Phi(x) > 0 \quad &\Longleftrightarrow \quad \forall\, x \in (D \backslash \overline{\Omega})
\end{aligned}$$

where D is a fixed reference domain in which all admissible shapes Ω are included, i.e. $\Omega \subset D$. In the level set method, the scalar function Φ can be specified in any specific form, and it is often described in a discrete counterpart. In most cases Φ is specified by a regular sampling on a rectilinear grid and constructed to be a signed distance function to the boundary. With such a signed distance function, i.e., $|\nabla\Phi| = 1$, many geometric properties of the boundary can be readily expressed, for instance the unit outward normal $\mathbf{n}$ of the boundary is given by $\mathbf{n} = \nabla\Phi$.

Propagation of free boundary of a structure during the optimization is described by the Hamilton-Jacobi equation:

$$\frac{\partial\Phi}{\partial t} + \mathbf{V} \cdot \nabla\Phi = 0 \tag{1}$$

where $\mathbf{V}$ is the velocity vector defined on the boundary Γ, as illustrated in Fig. 1. The velocity is an important link between the level set method and an optimization

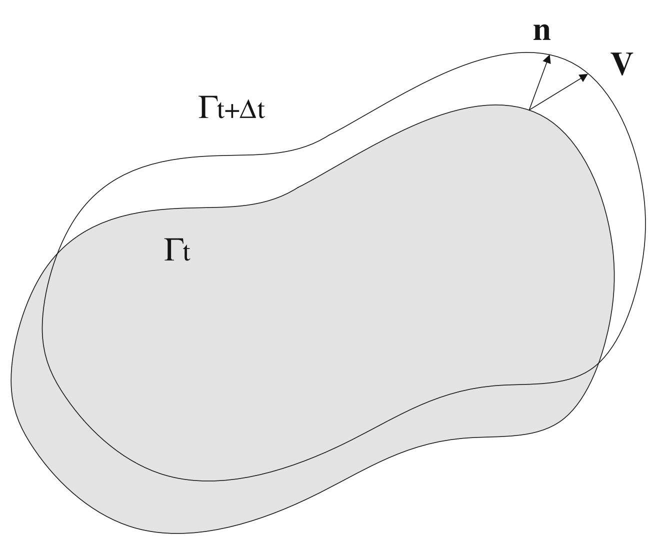

Fig. 1 The velocity and the propagation of a boundary

algorithm (Wang et al. 2003), and it is usually called the "design velocity". The level set based structure optimization method is a boundary variation method. During the course of optimization, variation of boundary shape described by the design velocity that improves the current design is obtained as a result of shape sensitivity analysis. Then, such boundary variation is treated as an advection velocity in the Hamilton-Jacobi equation of the level set method for updating the boundary of a structure. In other words, an optimization algorithm gives the desired variation of the free boundary, while the Hamilton-Jacobi equation performs the variation. A feature of the level set based method is that it essentially maintains the 0-1 nature of topology optimization (Xia and Wang 2008).

An important property of the Hamilton-Jacobi equation Eq. 1 is that it satisfies the maximum principle, and as a result voids can not be nucleated in the interior of a structure (Allaire et al. 2004). This makes the level set based topology optimization, especially in 2D, be sensitive to initial design (Allaire et al. 2004). To overcome this dependence much effort has been made to incorporate the topological derivative (Sokolowski and Zochowski 1999; Eschenauer et al. 1994; Céa et al. 2000) into the level set based method (Allaire and Jouve 2006; Burger et al. 2004; He et al. 2007). In 3D the situation is slightly different where although a new void cannot be nucleated right in the interior of a structure, a hole can be "tunneled" through the material region in between two pieces of boundary (Xia et al. 2010). Therefore, as compared to the situation in 2D, the level set based optimization in 3D is topologically more flexible and less sensitive to the initial design (Xia et al. 2010). However, in the context of optimization of a cast part, since it is required that a cast part should not have any interior void, the lack of void nucleation in the level set based method actually appears as an advantage for this specific application. Therefore, in the present study we use the conventional level set based method without using the topological derivative.

3 Parting direction and molding condition

In the present study, we consider the casting process that uses two molds. Examples of the cases where the molds can and cannot be removed are given in Fig. 2a and Fig. 2b, respectively. In Fig. 2a, the upper mold can be removed in the direction **d** and the lower one in the direction −**d** without colliding with the cast part. In Fig. 2b, the lower mold is stuck by a slot called undercut and cannot be removed in the direction −**d**. Besides the undercut, another requirement is that there should exist no interior void, i.e., a region completely contained in the interior of a structure.

The direction in which a mold is removed is called the parting direction, and the surface where the two molds contact each other is called the parting surface. Parting direction, parting surface, and undercut constitute the main parameters that significantly affects the mouldability and the manufacturing costs of a cast part. *For a designed structure*, the parting direction is usually obtained as the solution to an optimization problem so that certain criteria, e.g., number and volume of undercuts features, reach the optima (Weinstein and Manoochehri 1996, 1997; Nee et al. 1997; Fu et al. 1999), hence minimizing the manufacturing costs. *For the optimal design of a cast part in our present study, the parting direction affects not only the manufacturing costs but also the working performance*. In conventional optimal design of a cast part, the parting direction is pre-selected by a designer and fixed throughout the optimization. With such an approach, when the optimization is performed with a different parting direction, the resulting design will also be different, and more importantly it will end up with different working performance. In other words, the working performance depends on the parting direction. Therefore, it will be better if we take the parting direction as a design variable of the optimization of a cast part so that the working performance can be optimized as much as possible. The parting surface also needs careful consideration. Simple parting surface, preferably a plane, is often used because

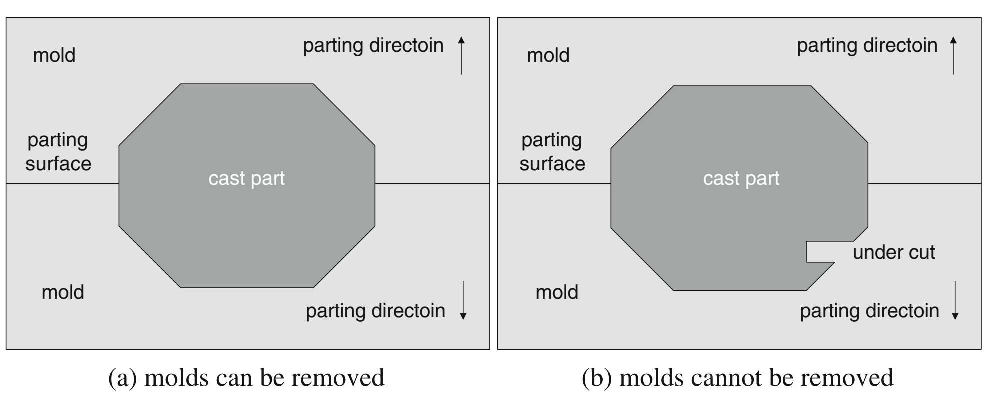

Fig. 2 Examples of the cases where the molds can be (**a**) and cannot be removed (**b**)

it reduces the costs of mold fabrication and the complexity of mold operation.

Suppose that a plane parting surface separates the boundary of a structure into two pieces(each piece staying in one mold). For boundary piece Γ^i ($i = 1, 2$), the molding constraint of casting process can be given as follows according to Chen et al. (1993); Fu et al. (2002):

$$\mathbf{d}^i \cdot \mathbf{n}(x) \geq 0, \quad \forall\, x \in \Gamma^i,\ i = 1, 2 \tag{2}$$

where $\mathbf{d}^i$ is the parting direction; $\mathbf{n}$ is the outward normal to the boundary. Equation 2 has an important role in the optimization of a cast part. On one hand, Eq. 2 gives an admissible set of parting direction for a fixed cast part, i.e., there may exist many admissible parting directions. On the other hand, Eq. 2 also gives a constraint on the shape of the boundary of a structure for a fixed parting direction.

Suppose that the optimization of a cast part starts with an initial design and an initial parting direction that satisfy the molding constraint Eq. 2. During the optimization, the boundary (hence the normal vector $\mathbf{n}$) and the parting direction $\mathbf{d}$ will be changed. But how can we ensure the resulting structure and parting direction still satisfy the molding constraint Eq. 2? This is the question to be answered.

Let us first see a molding condition on the design velocity. Suppose that a parting surface separates the boundary of a structure into two pieces Γ^i, $i = 1, 2$, and the mold for Γ^i can be parted in direction $\mathbf{d}^i$. When the boundary of a moldable structure(i.e., the molding constraint Eq. 2 is satisfied) is to be updated by a design velocity $\mathbf{V}$, the updated structure will be guaranteed to be moldable in direction $\mathbf{d}^i$ if the design velocity has the following form:

$$\mathbf{V}(x) = \lambda(x)\,\mathbf{d}^i, \quad \forall x \in \Gamma^i,\ i = 1, 2 \tag{3}$$

where λ is a scalar field along the boundary.

Equation 3 is the molding condition on the design velocity. According to Eq. 3 any point on the boundary of a structure is made to move in the parting direction $\mathbf{d}^i$ (when $\lambda(x) > 0$) or $-\mathbf{d}^i$ (when $\lambda(x) < 0$), and when $\lambda = 0$ the point does not move. Therefore, the motion of any point on the structure boundary is restricted to have no components in the directions orthogonal to the parting direction, thus it can not change a point that is visible from the parting directions to be invisible, which would make the structure unmoldable. According to Eq. 2, the molding condition on the design velocity is a sufficient (but not essentially a necessary) constraint.

According to the molding condition on the design velocity of the level set based structure optimization, we need to look for a scalar field λ and parting directions $\mathbf{d}^i$ in each iteration of the optimization, which is described in the next section.

4 Optimization of a cast part

In this section we take compliance as an example of the working performance of a cast part and present the definition of the optimization problem and its solution.

A shape $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) is an open bounded set occupied by isotropic linear elastic solid material. The Lipschitz continuous boundary of Ω consists of three disjoint parts

$$\partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_H$$

where a Dirichlet boundary condition is imposed on Γ_D, a Neumann boundary condition on Γ_N, and a homogeneous Neumann boundary condition, i.e., traction free, on Γ_H. In the present study, Γ_H is the only part subject to optimization and is free to move during the course of optimization.

The state equation of an elastic structure is

$$\begin{cases} -\mathrm{div}\,(\sigma(\mathbf{u})) = -\mathbf{f} & \text{in } \Omega, \\ \mathbf{u} = 0 & \text{on } \Gamma_D, \\ \sigma(\mathbf{u})\,\mathbf{n} = \mathbf{t} & \text{on } \Gamma_N. \end{cases} \tag{4}$$

where $\mathbf{u}$ is the displacement filed; $\sigma(\mathbf{u}) = A\,e(\mathbf{u})$ is the stress tensor; A describes the Hooke's law; $e(\mathbf{u})$ is the strain tensor; $\mathbf{f}$ is the body force; $\mathbf{t}$ is the traction force; and $\mathbf{n}$ is the unit outward normal to the boundary. The weak form of Eq. 4 is given by

$$a(\mathbf{u}, \mathbf{v}) = \ell(\mathbf{v}), \quad \forall \mathbf{v} \in \mathbf{U}$$

where $\mathbf{U}$ denotes the space of kinematically admissible displacement fields, and

$$\begin{aligned} a(\mathbf{u}, \mathbf{v}) &= \int_\Omega A\,e(\mathbf{u}) \cdot e(\mathbf{v})\,\mathrm{d}\Omega \\ \ell(\mathbf{v}) &= \int_\Omega \mathbf{f} \cdot \mathbf{v}\mathrm{d}\Omega + \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{v}\mathrm{d}\Gamma \end{aligned}$$

The minimum compliance optimization problem for a cast part is defined as:

$$\begin{aligned} \min_{\Omega,\mathbf{d}} \quad & \ell(\mathbf{u}) \\ \text{s.t.} \quad & a(\mathbf{u}, \mathbf{v}) = \ell(\mathbf{v}), \quad \forall \mathbf{v} \in \mathbf{U} \\ & \int_\Omega \mathrm{d}\Omega - \overline{V} \leq 0 \end{aligned} \tag{5}$$

where $\overline{V}$ is the upper bound of the volume of material allowed for the structure. The design variables in this optimization problem comprise the boundary shape of the cast part and the parting direction.

Assuming there exists no body force, i.e., $\mathbf{f} = 0$, and noticing that the only part of a structure's boundary subject to optimization is the traction free boundary Γ_H, the shape derivative is given by Allaire et al. (2004):

$$J' = \int_{\Gamma_H} G\,\mathbf{n} \cdot \mathbf{V}\,\mathrm{d}\Gamma, \quad G = l - A\,e(\mathbf{u}) \cdot e(\mathbf{u}) \tag{6}$$

where $G\mathbf{n}$ is the so-called shape gradient; G is the shape gradient density; l is a positive Lagrange multiplier. In the present work, we employ the augmented Lagrange multiplier method Nocedal and Wright (1999) to determine l. Substituting the molding condition on the design velocity Eq. 3 into Eq. 6, we get

$$J' = \sum_{i=1}^{2} \int_{\Gamma_H^i} G\,\mathbf{n}\cdot(\lambda\,\mathbf{d}^i)\,\mathrm{d}\Gamma \tag{7}$$

In order to minimize the objective function J, we need to choose a scalar field λ and parting directions $\mathbf{d}^i$ such that $J' \leq 0$. In fact, there are infinitely many choices of λ and $\mathbf{d}^i$. Among these choices, the simplest one is described as follows.

For the scalar field λ, it can be easily observed in Eq. 7 that if

$$\lambda = -G \tag{8}$$

we get

$$J' = -\sum_{i=1}^{2} \int_{\Gamma_H^i} G^2\,\mathbf{n}\cdot\mathbf{d}^i\,\mathrm{d}\Gamma \tag{9}$$

Since the parting direction $\mathbf{d}^i$ satisfy the molding constraint Eq. 2, the derivative J' is non-positive, i.e., $J' \leq 0$.

After choosing λ according to Eq. 8, we need to find parting directions $\mathbf{d}^1$ and $\mathbf{d}^2$ that simultaneously satisfies the molding constraint Eq. 2 and ensures J' given by Eq. 9 be non-positive. For this purpose, the steepest descent method is employed. The idea is to find a parting direction that induces the biggest descent of J, i.e., the smallest J'. In other word, the parting direction $\mathbf{d}^i$ is the solution to another optimization problem

$$\begin{array}{ll} \min\limits_{\mathbf{d}^i} & J' = -\left[\int_{\Gamma_H^i} G^2\,\mathbf{n}\,\mathrm{d}\Gamma\right]\cdot\mathbf{d}^i \\ \text{s.t.} & \mathbf{d}^i\cdot\mathbf{n}(x) \geq 0,\ \ \forall x \in \Gamma_H^i \\ & -1 \leq \mathrm{d}_x^i, \mathrm{d}_y^i, \mathrm{d}_z^i \leq 1 \end{array} \tag{10}$$

Problem (Eq. 10) is a liner optimization problem with three design variables $\mathrm{d}_x^i, \mathrm{d}_y^i, \mathrm{d}_z^i$ (the component of parting direction $\mathbf{d}^i$ along axis x, y, z) and a lot of inequality constraints. In the present work, since the level set function Φ is specified by a regular sampling on a rectilinear grid, we can compute a normal vector and impose an inequality constraint $\mathbf{d}^i\cdot\mathbf{n}(x) \geq 0$ on each cell of the grid cut by the free boundary Γ_H.

The inequality constraint $\mathbf{d}^i\cdot\mathbf{n}(x) \geq 0$ in Eq. 10 is just a reminiscent of the molding constraint Eq. 2 and defines an admissible set of parting direction for the current structure. However, these inequalities only constrain the direction of vector $\mathbf{d}^i$, and the length of vector $\mathbf{d}^i$ is not constrained. Therefore, the space depicted by $\mathbf{d}^i\cdot\mathbf{n}(x) \geq 0$ is indeed unbounded, which would make the optimization problem Eq. 10 have no solution. To overcome this problem, the inequality constraint $-1 \leq \mathrm{d}_x^i, \mathrm{d}_y^i, \mathrm{d}_z^i \leq 1$ is added to the optimization to make sure the admissible set of the vector $\mathbf{d}^i$ be bounded. Other inequality constraints can also be used for this purpose, for instance $\sqrt{(\mathrm{d}_x^i)^2 + (\mathrm{d}_y^i)^2 + (\mathrm{d}_z^i)^2} \leq 1$, as long as the direction of the vector $\mathbf{d}^i$ is not constrained by it but the length of $\mathbf{d}^i$ is bounded by it. In our view, the length constraint in Eq. 10 is the simplest one.

Moreover, there also exist other choices of λ. For example, one may choose $\lambda = -G\mathbf{n}\cdot\mathbf{d}^i$ which is the steepest descent direction of λ. But such choice will lead to a nonlinear objective function $-\int_{\Gamma_H^i}\left[G\,\mathbf{n}\cdot\mathbf{d}^i\right]^2\mathrm{d}\Gamma$ for optimizing $\mathbf{d}^i$, which is much more complicated than the linear one given by Eq. 10. Therefore, in the present work, we choose λ and $\mathbf{d}^i$ according to Eqs. 8 and 10.

Now, we can see how the simultaneous optimization of cast part and parting direction works. In each iteration, an optimal parting direction is first computed for the current structure, then the boundary of the current structure is updated by a design velocity that guarantees the updated design be moldable with the optimal parting direction. Therefore, although the parting direction may be changed during the optimization, the structure will be always moldable in the current parting direction. After the optimization, the parting surface of the cast part can be selected to be a plane that is perpendicular to the parting direction or can be computed by the methods developed for computer aided mold design, and the reader is refereed to Li (2007) and the references therein.

5 Numerical implementation

5.1 Implementation of level set

It should be noted that in the level set based method the design velocity $\mathbf{V}$ defined on the free boundary of a structure must be extended to the whole design domain D or a narrow band around the free boundary (Osher and Sethian 1988; Sethian 1999). In the optimization of continuum structures, since a fixed finite element mesh is used for the finite element analysis, and voids are mimicked by artificial weak material, the design velocity is naturally extended to the entire design domain D as $\mathbf{V}^e = \mathbf{V},\ \forall x \in D$. This extended velocity, however, will introduce discontinuity of the velocity at the neighborhood of the traction free boundary since the strain field is not continuous across the boundary. To guarantee a smooth propagation of the free boundary, this discontinuity should be eliminated. Hence, a smoothing operation is often introduced for the velocity extension (Burger 2003; Wang and Wang 2005; Gournay 2006), and in the present work we use the method presented in Wang and Wang (2005).

The HJ Eq. 1 is a hyperbolic type of PDE (Sethian 1999; Osher and Fedkiw 2002). A variety of spatial and time discretization schemes were devised to solve this type of PDE. In the present study, the first order upwind spatial differencing and forward Euler time differencing are utilized. Finally, since the level set function Φ often becomes too flat or too steep during optimization which leads to increasing numerical error, a reinitialization procedure is periodically performed to restore Φ to a signed distance function to the structural boundary, i.e., to restore $|\nabla\Phi| = 1$. More details of the numerical computations of level set can be found in Sethian (1999); Osher and Fedkiw (2002).

5.2 Finite element method

In the present study, an Eulerian-type method employing a fixed mesh and artificial weak material is adopted as the finite element analysis (FEA) tool(Allaire et al. 2004; Wang et al. 2003). In this method, instead of solving the state equation on structure Ω, we solve it on the entire design domain D with the void $D\backslash\Omega$ being represented by weak material. The material properties of the weak material is tailored so that the results of FEA obtained on the entire design domain D is consistent to that obtained on the structure $\Omega \subset D$.

5.3 Optimization procedure

A detail description of our optimization procedure is given below:

Step 1: Initialize the level set function Φ. Initialize all the parameters.
Step 2: Solve the linear elastic equation and compute the shape gradient density G.
Step 3: Optimize the parting direction.
Step 4: Update Φ according to Hamilton-Jacobi equation for several time steps. Reinitialize Φ.
Step 5: Update Lagrange multiplier ℓ.
Step 6: Check whether the optimization converges. If not, repeat Steps 2 through 5 till convergence.

6 Numerical examples

In this section the proposed level set based method is applied to several examples in three dimensions. In these examples, it is assumed that the solid material has a Young's modulus $E = 1$ and Poisson's ratio $\nu = 0.3$, the weak material has a Young's modulus $E = 0.001$ and Poisson's ratio $\nu = 0.3$. Moreover, only one parting direction is optimized in these examples, and the other parting direction is selected to be opposite to the optimized one.

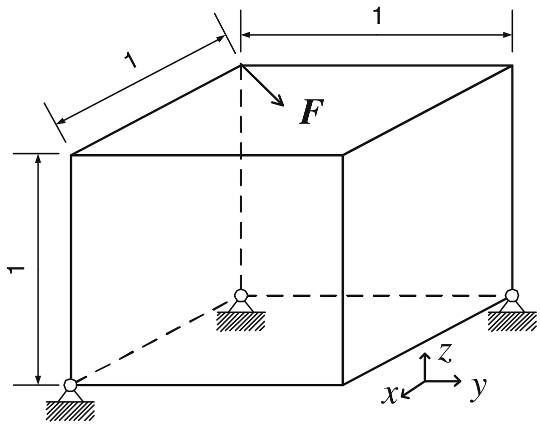

Fig. 3 Design problem of a short cantilever beam

6.1 Example 1

The optimal design problem is shown in Fig. 3. The whole design domain D is a cube of size $1\times1\times1$ with the three fixed corners. A point load $F = (1, 1, 0)$ is applied at a corner. The volume allowed for the structure is 20% of the design domain.

First, the design problem is solved without considering molding constraint. Second, it is solved using the method proposed in our previous study Xia et al. (2010) that integrated the molding constraint into the structure optimization. Finally, the design problem is solved using the method proposed in the present study that simultaneously optimize a cast part and parting direction. In the last optimization, the initial Φ is a signed distance function to three faces (whose normal direction is along the positive direction of corresponding axis) of the design domain. In other word, although the other faces are indeed a boundary of the initial design, it is not considered as the zero level set of the initial Φ.

The results are shown in Fig. 4. We can see that the structure obtained by the simultaneous optimization is similar to the one obtained without considering the molding constraint and is quite different to the one obtained with pre-selected and fixed parting direction. In fact, the simultaneous optimization is more flexible as compared to the optimization with fixed parting direction. The convergence history of the simultaneous optimization is shown in Fig. 5 from which we can see that of optimization converges smoothly.

From the results shown in the caption of Fig. 4, we can see that the compliance of the structure obtained without considering molding constraint is the smallest, and

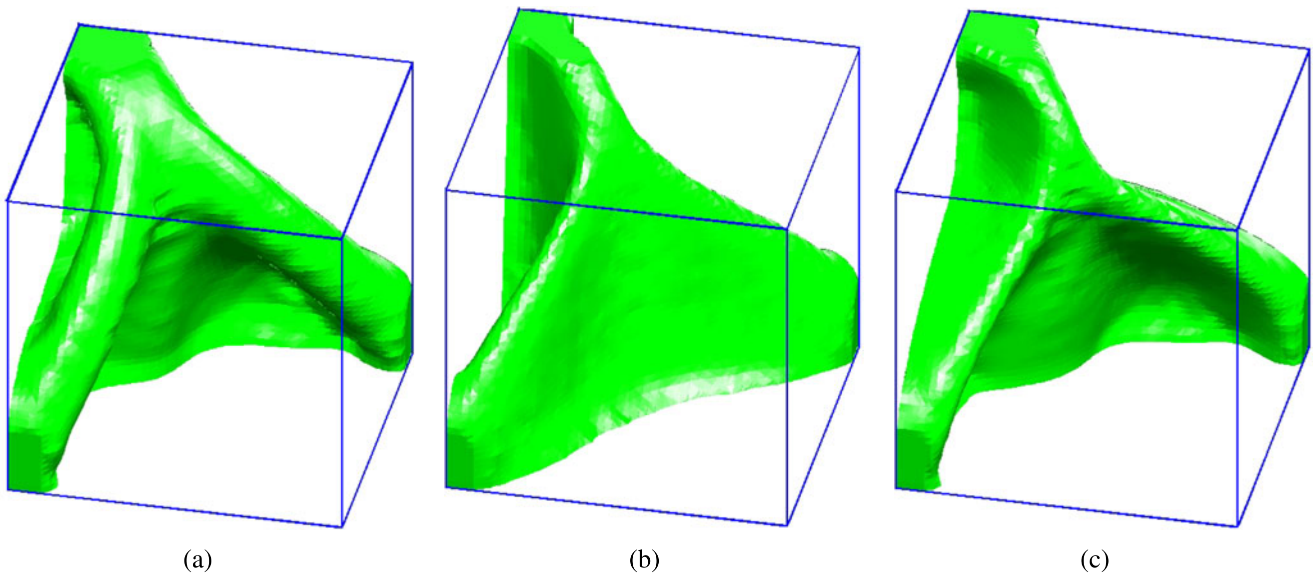

Fig. 4 The optimal structures. **a** without considering molding constraint, the compliance is 452; **b** with molding constraint (parting direction is pre-selected to be along z axis, parting surface is pre-selected to be the bottom of the cube), the compliance is 502; **c** with molding constraint (parting direction is not pre-selected but simultaneously optimized), the compliance is 500, the optimized parting direction is (0.6, 0.6, 0.5)

that the compliance obtained by simultaneous optimization is slightly smaller than that obtained with fixed parting direction. In fact, the compliance of the optimal structure increases due to the molding constraints. When the molding constraint is incorporated into the structural optimization, whether it is solved via the SIMP method or the level set method, the flexibility for delivering topology changes during the optimization will be reduced. In other word, the molding constraint shrinks the design space and may exclude the optimal solution that would be obtained without the molding constraint. In this context the topologies are generally different from that obtained without molding constraint.

There also exist disadvantages of the present level set based method. As we can see from the examples, the optimization appears as a shape optimization because the design velocity has to comply with the molding condition. It is known that void can not be nucleated by the level set method during the course of optimization. This fact motivated the integration of topological derivative (Sokolowski and Zochowski 1999) into the level set based topology optimization (Allaire and Jouve 2006; Burger et al. 2004; He et al. 2007). With such integration, the optimization can introduce voids at the positions indicated by topological derivative to decrease the objective function. If the topological derivative is not used, one usually starts the level set-based optimization with an initial design having a lot of voids. These voids can gradually be removed by the level set method, and this case is also a topology optimization. However, in the present level set based optimization of cast part, since it does not integrate the topological derivative, neither allow the initial design to have internal voids, it is actually a shape optimization, and the results obtained are only local optima.

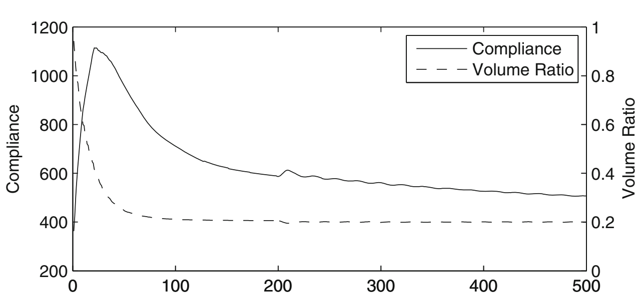

Fig. 5 Convergence history of simultaneous optimization

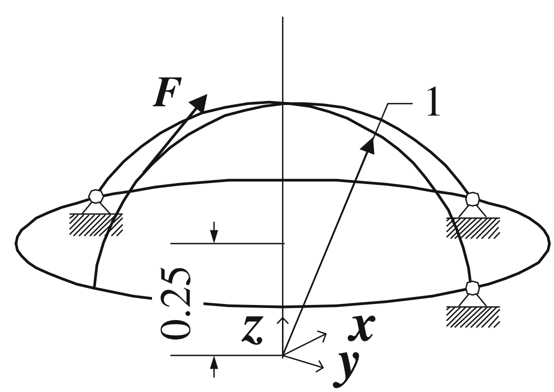

Fig. 6 Design problem of example 2

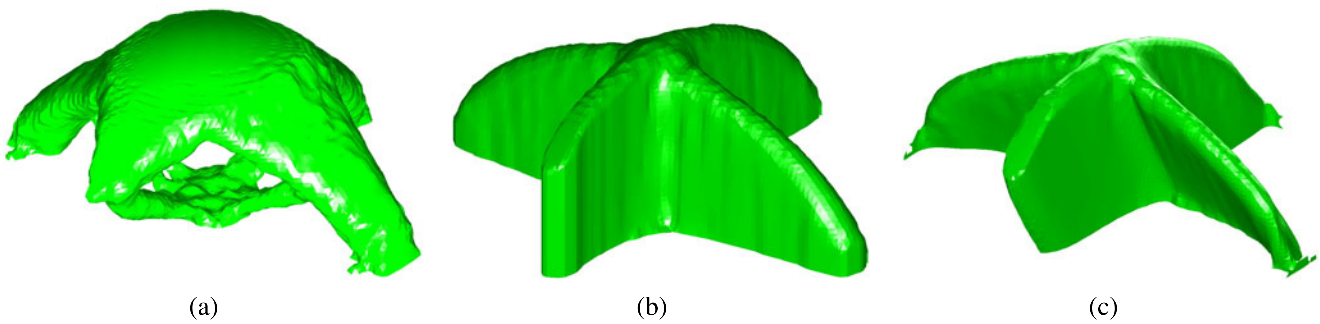

Fig. 7 The optimal structures. **a** without considering molding constraint, the compliance is 258; **b** with molding constraint (parting direction is pre-selected to be along z axis, parting surface is pre-selected to be the bottom of the spherical cap), the compliance is 430; **c** with molding constraint (parting direction is not pre-selected but simultaneously optimized), the compliance is 456, the optimized parting direction is (0.44, 0.00, 0.89)

6.2 Example 2

The optimal design problem is shown in Fig. 6. The whole design domain D is a spherical cap; the radius of the sphere is 1 and the height of the cap is 0.75. Three points (0.96, 0, 0), (0,0.96,0), (0,−0.96,0) in the bottom face are fixed, and the neighborhood around these points are treated as non-designable domain. A point load $F = (-1, 0, 1)$ is applied at a point (0.707, 0, 0.457) on the sphere. The volume allowed for the structure is 20% of the design domain.

First, the design problem is solved without considering molding constraint. Second, it is solved using the method proposed in our previous study (Xia et al. 2010) that integrated the molding constraint into the structure optimization. Finally, the design problem is solved using the method proposed in the present study that simultaneously optimize a cast part and parting direction. In the last two optimizations, the initial Φ is a signed distance function to the spherical face of the design domain. In other words, although the bottom face is indeed a boundary of the initial design, it is not considered as the zero level set of the initial Φ. Moreover, in the last optimization, due to the symmetry of the design domain, the y-component in the parting direction is not considered in the optimization of parting direction. The results are shown in Fig. 7. We can see that the structure obtained by the simultaneous optimization is similar to the one obtained with pre-selected and fixed parting direction, but with larger compliance. In our view, this is because the optimal parting direction depends on the initial design. The convergence history of the simultaneous optimization is shown in Fig. 8 from which we can see that of optimization converges smoothly. The bump in the plot of convergence history is because the augmented Lagrange multiplier method is applied after the 200-th iteration.

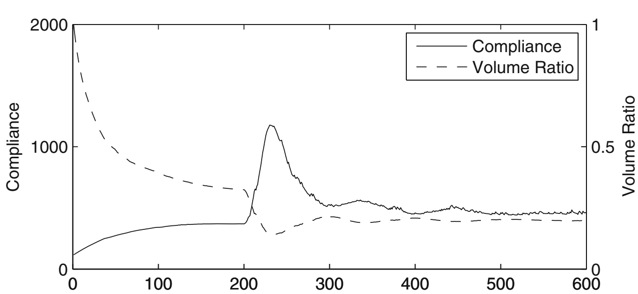

Fig. 8 Convergence history of simultaneous optimization

7 Conclusion

This paper presents a level set based method for the simultaneous optimization of cast part and parting direction. In each iteration, an optimal parting direction is first computed for the current structure, then the boundary of the current structure is updated by a design velocity that guarantees the design be moldable with the optimal parting direction. Therefore, although the parting direction may be changed during the optimization, the structure will be always moldable in the current parting direction. Numerical examples are provided in 3D.

Acknowledgments This research work is partly supported by the Fundamental Research Funds for the Central Universities, HUST, (Grant No. Q2009009), the Natural Science Foundation of Hubei province (Grant No. 2009CDB321), the National Fundamental Research Programme of China (Grant No. 2009CB724204), which the authors gratefully acknowledge. The insightful comments of the reviewers' are cordially appreciated.

References

Allaire G, Jouve F (2006) Coupling the level set method and the topological gradient in structural optimization. In: Bendsøe MP, Olhoff, N, Sigmund O (Eds) IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials, vol 137. pp 3–12

Allaire G, Jouve F, Toader AM (2002) A level-set method for shape optimization. C.R., Acad, Sci. Paris, Serie I 334:1–6

Allaire G, Jouve F, Toader AM (2004) Structural optimization using sensitivity analysis and a level-set method. J Comput Phys 194:363–393

Baumgartner A, Harzheim L, Mettheck C (1992) SKO (Soft Kill Option): the biological way to find an optimum structure topology. Int J Fatigue 14:387–393

Bendsøe MP (1989) Optimal shape design as a material distribution problem. Struct Optim 1:193–202

Bendsøe MP, Sigmund O (2003) Topology Optimization: Theory, Methods and Applications. Springer-Verlag, Berlin

Burger M (2003) A framework for the construction of level set methods for shape optimization and reconstruction. Interfaces Free Bound 5:301–329

Burger M, Hackl B, Ring W (2004) Incorporating topological derivatives into level set methods. J Comput Phys 194:344–362

Céa J, Garreau S, Guillaume P, Masmoudi M (2000) The shape and topological optimizations connection. Computer Methods in Applied Mechanics and Engineering. 188:713–726

Chen LL, Chou SY, Woo TC (1993) Parting directions for mould and die design. Computer-Aided Design 25:762–768

Eschenauer HA, Kobelev HA, Schumacher A (1994) Bubble method for topology and shape optimization of structures. Struct Optim 8:142–151

Fu MW, Fuh JYH, Nee, AYC (1999) Generation of optimal parting direction based on undercut features in injection molded parts. IIE Trans 31:947–955

Fu MW, Nee AYC, Fuh JYH (2002) The application of surface visibility and moldability to parting line generation. Journal of Computer-Aided Design 34:469–480

Gournay FD (2006) Velocity extension for the level-set method and multiple eigenvalues in shape optimization. SIAM J Control Optim 45:343–367

Harzheim L, Graf G (1995) Optimization of engineering components with the sko method. In: the ninth international conference on vehicle structural mechanics and CAE, pp 235–243

Harzheim L, Graf G (2002) Topshape: an attempt to create design proposals including manufacturing constraints. Int J Veh Des 28:389–408

Harzheim L, Graf G (2005) A review of optimization of cast parts using topology optimization: I—topology optimization without manufacturing constraints. Struct Multidiscipl Optim 30:491–497

Harzheim L, Graf G (2006)A review of optimization of cast parts using topology optimization: II—topology optimization with manufacturing constraints. Struct Multidiscipl Optim 31:388–399

He L, Kao CY, Osher S (2007) Incorporating topological derivatives into shape derivative based level set methods. J Comput Phys 225:891–909

Leiva JP, Watson BC, Kosaka I (1999) Modern structural optimization concepts applied to topology optimization. In: 40th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference. St. Louis, MO

Leiva JP, Watson BC, Kosaka I (2004) An analytical directional growth topology parameterization to enforce manufacturing requirements. In: 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Material Conference. Palm Springs, CA

Leiva JP, Watson BC, Kosaka I (2004) An analytical bi-directional growth parameterization to obtain optimal castable topology designs. In: 10th AIAA/ISSMO symposium on multidisciplinary analysis and optimization. Albany, NY

Li WS, Martin RR, Langbein FC (2007) Generating smooth parting lines for mold deisgn for meshes. In: ACM symposium on Solid and Physical Modeling. Beijing, China, pp 193–204

Mattheck C (1990) Design and growth rule for biological structures and their application in engineering. Fatigue Fract Eng Mater Struct 13:535–550

Nee AYC, Fu MW, Fuh JYH, Lee KS, Zhang YF (1997) Determination of optimal parting direction in plastic injection mold design. annals of the. Annals of CIRP 46:429–432

Nocedal J, Wright SJ (1999) Numerical Optimization. Springer

Osher S, Fedkiw R (2002) Level Set Methods and Dynamic Implicit Surfaces. Springer-Verlag, New York

Osher S, Santosa F (2001) Level-set methods for optimization problems involving geometry and constraints: frequencies of a two-density inhomogeneous drum. J Comput Phys 171:272–288

Osher S, Sethian JA (1988) Front propagating with curvature dependent speed: Algorithms based on Hamilton-Jacobi formulations. J Comput Phys 78:12–49

Rozvany GIN, Zhou M, Birker T (1992) Generalized shape optimization without homogenization. Struct Optim 4:250–254

Sethian JA (1999) Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science. Cambridge Monographs on Applied and Computational Mathematics, 2nd edn. Cambridge University Press, Cambridge, UK

Sethian JA, Wiegmann A (2000) Structural boundary design via level set and immersed interface methods. J Comput Phys 163:489–528

Sokolowski J, Zochowski A (1999) On the topological derivative in shape optimization. SIAM J Control Optim 37:1251–1272

Wang MY, Wang XM, Guo DM (2003) A level set method for structural topology optimization. Comput. Methods Appl Mech Eng 192:227–246

Wang SY, Wang MY (2005) A moving superimposed finite element method for structural topology optimization. Comput. Methods Appl Mech Eng 65:1892–1922

Weinstein M, Manoochehri S (1996) Geometric influence of a molded part on the draw direction range and parting line locations. J Mech Des 118:29–C39

Weinstein M, Manoochehri S (1997) Optimum parting line design of molded and cast parts for manufacturability. Journal of manufacturing Systems 16:1–12

Xia Q, Shi TL, Wang MY, Liu SY (2010) A level set based method for the optimization of cast part. Comput. Methods Appl Mech Eng 41:735–747

Xia Q, Wang MY (2008) Topology optimization of thermoelastic structures using level set method. Comput Mech 42:837–857

Zhou M, Shyy YK, Thomas HL (2001) Topology optimization with manufacturing constraints. In: 4th world congress of structural and multidisciplinary optimization. Dalian, China