ANT COLONY ALGORITHM FOR LAYOUT DECOMPOSITION IN DOUBLE/MULTIPLE PATTERNING LITHOGRAPHY

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ABSTRACT

Double or multiple patterning (DP/MP) lithography is an alternative for the sub-20 nm node and beyond. In MP, it is essential to solve a minimal patterning number to decompose the dense features. While in DP, it is required to remove odd conflict cycles with minimal stitch number. In this work, we apply an ant colony algorithm to address these two issues, respectively.

I. INTRODUCTION

As critical dimension further shrinks, double/multiple patterning (DP/MP) lithography is considered as one of the next generation solution for sub-20/14 nm nodes. DP/MP lithography involves decomposing a dense circuit pattern into a series of separate exposure and etch steps, and therefore the lithography resolution is improved [1-4].

In DP/MP, mask assignment is a key problem, which is essentially a two or multiple-color problem [1]. In this problem, the layout is represented with a conflict graph, and it is common to form odd conflict cycles. There are generally two way to remove these odd conflict cycles, one is MP whose goal is to solve a minimal number to color the dense feature, and the other is to remove conflicts with minimal stitch number using DP decomposition. It has been proved that these coloring problems belong to NP-hard problems [2]. Several works have been proposed on layout coloring using integer linear programming (ILP) [1, 3, 4]. However, since ILP is an exact algorithm, it is confined to solve small size coloring problem and therefore becomes infeasible when problem size increases.

In this work, we employ an ant colony algorithm (ACA) to perform layout coloring. ACA is a well-known meta-heuristic algorithm, in which a colony of artificial ants cooperates in exploring good solution for graph coloring problems [5]. It is suitable approach for solving hard combinatorial optimization problems [5,6].

II. THEORY

Conflict Graph and Stitch

Conflict graphs can be constructed based on a given layout and minimum same-color spacing [4]. The conflict graph Fig. 1(b) is an undirected graph with a set of nodes $\mathbf{P}^{(N)}$ and conflict edge \mathbf{E} . Each node in set $\mathbf{P}^{(N)}$ represents a polygon in layout Fig. 1(a). There is an edge in \mathbf{E} if and only if the two polygons are within minimum same-color spacing.



Figure 1: (a) The layout. (b) The corresponding conflict graph, where red nodes represent the polygons in (a) and blue solid line between nodes denote conflict.

The splitting of a polygon into multiple parts is referred as stitching [4]. In particular, stitch is used to break odd conflict cycle (e.g., in Fig. 1(b), p_1 , p_8 , p_9) into even cycle to get rid of coloring conflicts in DP.

Layout Decomposition by Envelope Polygon

Previous works on DP coloring mainly differ in the way stitches are performed, such as rule-based stitching method, rectangle segmentation method [1], and node projection algorithm [2-4]. In this work, we propose a new envelope technique. As Fig. 2 showed, polygon1 is the original one, envelope polygon is constructed based on minimum same-color spacing from its boundaries. According to the envelope polygon, it can efficiently clip overlap polygon into two polygons [7, 8].



Figure 2: The schematic of layout decomposition by envelope polygon.

Problem Formulation

Given a layout, its conflict graph is constructed and denoted by $G = (\mathbf{P}^{(N)}, \mathbf{E})$, where $\mathbf{P}^{(N)}$ is a set of nodes and \mathbf{E} is the matrix of edge. The set $\mathbf{P}^{(N)}$ contains *N* elements in total and each element is denoted as p_i , as shown in Fig. 1.

In multiple patterning (MP) without stitches inserted, all nodes in $\mathbf{P}^{(N)}$ should be colored according to \mathbf{E} and there are no conflicts within one color class set. The MP problem can be formulated as:

$$k^{*} = \arg\min_{k} (\mathbf{P}^{(N)} = \bigcup_{m=1}^{k} C_{m}),$$

s.t.:
$$\begin{cases} \forall m \neq n : C_{m} \cap C_{n} = \Phi \\ \forall i, j \in C_{m}, i \neq j : e_{i,j} \neq 1 \end{cases}$$
 (1)

Here, p_i is the *i*th node in set $\mathbf{P}^{(N)}$, and C_m is the *m*th color class set in which nodes are assigned to the *m*th color, $E_{i,j}$ in edge matrix \mathbf{E} is equal to 1 if and only if the two nodes p_i and p_j is conflicted, k is the total color number, and k^* is the solution of the MP problem. The object of this problem is to find minimal total color number k.

In DP, stitches are inserted to remove odd conflict cycles. According to envelope technique, a new conflict graph is constructed and denoted by $G_{\text{new}} = (\mathbf{P}_{\text{new}}^{(N)}, \mathbf{E}_{\text{new}})$. Then, all nodes in set $\mathbf{P}_{\text{new}}^{(N)}$ are assigned into two colors according to the edge matrix \mathbf{E}_{new} . The DP problem is formulated as:

$$N^{*} = \arg\min_{N'} \left(\mathbf{P}_{new}^{(N')} = \bigcup_{m=1}^{2} C_{m} \right),$$

s.t.:
$$\begin{cases} C_{1} \cap C_{2} = \Phi \\ \forall i, j \in C_{m}, i \neq j : e_{i,j}^{new} \neq 1 \end{cases}$$
 (2)

Here, the set $\mathbf{P}_{new}^{(N)}$ contains N' element in total and p_i^{new} denotes *i*th node in the new conflict graph. It is noted that the total stitch number is equal to N' - N. So, the object of DP problem is to seek a minimal number N^* .

III.ANT COLONY ALGORITHM

Ant colony algorithm (ACA), as a meta-heuristic algorithm, is applied to solve the layout conflict graph coloring problems. In ACA, at each stage l, pheromone trails $\tau_{l,m}$ are update by the (l-1) ants that have produced solutions. It represents that colony experience of selecting nodes p_i into coloring class C_m is update as:

$$\tau_{i,m} = (1 - \rho)\tau_{i,m} + \sum_{n=1}^{l-1} \Delta \tau_{i,m}(n).$$
(3)

Here ρ is the evaporation rate of pheromone trails, and $\Delta \tau_{i,m}(n)$ is the quantity of pheromone tails deposited on the nodes p_i and class C_m by ant n.

During the process of constructing a solution, the probabilistic $\Psi_{i,m}^{l}$ is defined to determine which uncolored node p_i to be added into the color class set C_m as [7]:

$$\Psi_{i,m}^{l} = \frac{\tau_{i,m}^{\alpha} \eta_{i,m}^{\beta}}{\sum_{i \in W^{m}} \tau_{i,m}^{\alpha} \eta_{i,m}^{\beta}}, \ i \in W^{m},$$
(4)

where heuristic information $\eta_{i,m}$ measures the desirability of putting nodes p_i into the color class set C_m , W^m contains all the nodes that have no conflicts with the nodes already in color class set C_m , α and β determine the relative importance of the pheromone trails verse heuristic information.

ACA for Multiple Patterning

In multiple patterning problem Eq. (1), the heuristic information $\eta_{i,m}$ is defined as:

$$\eta_{i,m} = \deg(i), \, i \in W^m, \tag{6}$$

where function deg(*i*) calculates the degree of nodes p_i in the graph as showed in Ref. [8]. And the $\Delta \tau_{i,m}(n)$ in Eq. (3) is defined as:

$$\Delta \tau_{i,m}(n) = \frac{1}{k}.$$
(7)

ACA For Double Patterning

In double pattern problem Eq.(2), we first clip necessarily polygon into multiple ones according to the envelope technique. Different from MP problem without stitches inserted, heuristic information $\eta_{i,m}$ is defined as:

$$\eta_{i,m} = \frac{Q_1}{f(N_l')},\tag{8}$$

where parameter Q_1 is a constant, function $f(N_i)$ is to get the number of stitch in this step and denoted as $f(N_i) = N_i$ -N. The N_i is the number of nodes after uncolored node p_i being added into the color class C_m at stage l.

Also the deposited pheromone at stage n, $\Delta \tau_{i,m}(n)$, here is defined as:

$$\Delta \tau_{i,m}(n) = \frac{Q_2}{g(N')}.$$
(9)

Here Q_2 is a constant, function g(N) = N - N is to acquire the stitch number used by the ant *n*. The N is the node number after ant *n* finished coloring.

The flowchart of ACA for DP problem is showed in Fig. 3.



Figure 3: Flowchart of ACA DP layout decomposition.

IV. SIMULATION RESULT

In ACA, we set the adjusted parameters $\alpha = 2.0$, $\beta = 4.0$, $Q_1 = Q_2 = 1.0$, and $\rho = 0.5$. The colony size is set to 30 ants. The same-color spacing is set as two times of its

critical dimension.

Results of Multiple Patterning

As Fig. 4(a) showed, the layout contains 12 polygons which is denoted by $\mathbf{P}^{(12)}$. The matrix \mathbf{E} contains 12×12 and its each element $e_{i,j}$ represents the conflict between polygon p_i and p_j . Figure 4(b) shows the coloring result, where polygons in same color are assigned into the same mask. The class set $C_1 = \{p_1, p_3, p_7, p_{10}\}$ is colored in blue, $C_2 = \{p_2, p_4, p_9, p_{11}\}$ in green, and $C_3 = \{p_5, p_6, p_8, p_{12}\}$ in red. The minimal total coloring number k is 3 in this case.



Figure 4: Example of multiple patterning.

Results of Double Patterning

The original layout Fig. 5(a) contains N = 12 polygons which is represented by $\mathbf{P}^{(12)}$ and the conflict graph is shown in Fig. 5(b).



Figure 5: Example of layout and its conflict graph.

After clipping polygons into multiple ones with envelope technique, the layout denoted by $\mathbf{P}^{(12)}$ is updated to $\mathbf{P}_{mew}^{(38)}$ which contains 38 polygons as showed in Fig 6(a). Then ACA is applied to layout coloring, and the result is showed in Fig. 6(b). The cyan and magenta color are respectively used to denotes the two decomposition layout. In this case, class set $C_1 = \{p_1, p_6, p_7, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{20}, p_{21}, p_{22}, p_{23}, p_{24}, p_{27}, p_{28}, p_{33}, p_{34}, p_{36}\}$ is colored in magenta, and $C_2 = \{p_2, p_3, p_4, p_5, p_8, p_{16}, p_{17}, p_{18}, p_{19}, p_{25}, p_{29}, p_{30}, p_{31}, p_{32}, p_{35}, p_{37}, p_{38}\}$ in cyan. The touched clipped polygons are merged when assigned into a same color class set. As a result, the final node number N^r turn to be 22. So the stitch number g(N') = N' - N = 22 - 12 = 10.

V. CONCLUSION

We applied an ant colony algorithm method to solve the multiple and double patterning problem. Simulation results demonstrate the validity of the proposed algorithm.



Figure 6: Example of double pattering.

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