Comment on "Three-dimensional imaging of a phase object from a single sample orientation using an optical laser"

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A recent article by Chen *et al.* [Phys. Rev. B **84**, 224104 (2011)] purports a "matrix rank analysis" and an optical experiment in support of the three-dimensional (3D) imaging technique called "ankylography." However, the mathematical analysis does not appear to be conclusive, and the one used in the experiment is more a 3D-supported scattering object of actually 2D complexity than a 3D-distributed scattering object of truly 3D complexity. Consequently, the article provides little support to the "ankylography" technique.

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In a recent article,[1] Chen *et al.* claimed to have provided a "matrix rank analysis" and an optical experiment of coherent diffraction imaging in support of the so-called "ankylography" technique of lensless imaging.[2] Here we point out that the matrix equations in Ref. 1 do not constitute a conclusive analysis, and the scattering object chosen for the "ankylography experiment" is not a typical object with three-dimensional (3D) complexity to test the purported technique of "reconstruction of a 3D object from a single sample orientation."

In the "analytical" section of Ref. 1, the authors started with discretizing a 3D object distribution $\rho(x,y,z)$ into a vector X and its far-field diffraction amplitude $F(k_x,k_y,k_z)$ into another vector A. Then a matrix B was written down explicitly to represent the operation of far-field diffraction. The matrix equation $BX = A$ is already analyzable by the well-known technique of singular value decomposition. But the authors introduced redundancy to turn the linear system into $B'X' = A$ with B' being a square matrix. The sum of the presented equations amount to reformulating the equation of far-field diffraction

$$F(k_x,k_y,k_z) = \int \rho(x,y,z)\exp[-i2\pi(k_x x + k_y y + k_z z)]dx\,dy\,dz$$

into a matrix equation $BX = A$ or $B'X' = A$. The authors did not proceed to perform a spectral analysis on the matrix B or B'. Instead, they provided two numerical examples, one of which has X sized $7^3 \times 1$ and A sized 785×1, when it is numerically checked that the 785×785 matrix B' has 785 ($>7^3$) significant singular values with respect to a "tolerance" of 10^{-3}, while the other has X sized $14^3 \times 1$ and A sized $(\sim 11\,300) \times 1$, when it is observed that the tolerance needs to be lowered down to 10^{-6} in order to have as many usable singular values as the size of X. The exact definition of tolerance is nowhere to be found. A clear specification of the "signal-to-noise ratio" or tolerance to matrix singular values goes a long way toward better communication. Nevertheless, independent of a specific interpretation, the precipitous decay of singular values as the authors observed and reported is alarming, which seems to indicate more often rapid exponential dying down than gentle polynomial decreasing. Indeed, there has been controversy[3–5] surrounding the proposed method of ankylography, much of which has to do with the asymptotic scaling of the singular values.

On the experimental side, the authors recorded a high-resolution diffraction pattern of a 3D-supported weak phase object, performed an "ankylographic" reconstruction of the object using their "back and forth iteration" method, and claimed to have experimentally demonstrated the ankylography approach of 3D object reconstruction or 3D imaging. However, the experiment seems to have confused a 3D-supported scattering object of actually 2D complexity with a 3D-distributed scattering object of truly 3D complexity. Although the employed phase object is 3D supported, namely, occupying a 3D volume, its structure is too simple to represent a general 3D distribution of a scattering object, which has 3D complexity and exhibits variations inside the support volume. By contrast, the weak phase object in question remains constant inside its support volume, and belongs to a class of sparse 3D objects with 2D complexity, in the sense that the number of unknowns of an object grows quadratically as its three dimensions are scaled up simultaneously by the same factor. Through the theory and methods of compressed sensing,[6,7] it is well established and known that such sparse objects may be accurately reconstructed by a number of measurements that scales slower than their support volume. Should the ankylography technique be limited to such sparse 3D objects, such limitation needs to be stated clearly, not in fuzzy terms like "under certain circumstances" and "certain classes of samples." That the concerned weak phase object is sparse and may be accurately reconstructed from a 2D diffraction measurement can be easily seen by introducing differential operators $(\partial/\partial x, \partial/\partial y, \partial/\partial z)$ to the scattering object, which transform the scattering response to an explicitly sparse signal field having nonzero entries only at the boundary of the support volume of the original object. In practice, the differential operators may be realized by displacing the scattering object slightly and recording the changes of a holographic diffraction pattern, or equivalently, multiplying the recorded wave amplitudes of a holographic diffraction pattern by (k_x,k_y,k_z).

In conclusion, we believe that Ref. 1 falls short in providing "a matrix rank analysis to explain why ankylography under certain circumstances can be used to determine the 3D structure from a single sample orientation" and demonstrating "this approach (to determine the 3D structure from a single sample orientation) by performing an ankylography experiment." As a consequence, the article provides little support to the ankylography technique.

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