Mueller matrix polarimeter with imperfect compensators: calibration and correction

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Abstract: We propose a method to calibrate the depolarization parameters of the imperfect compensators in dual-rotating compensator Mueller matrix polarimeters, and deduce a set of correction equations for the Mueller matrix calculation.
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1. Introduction

Recently, the Mueller matrix polarimeter (MMP) based on the coupled ferroelectric liquid crystal cell [1,2] and/or dual-rotating compensators [3,4] has been developed and applied as a powerful tool for the characterization of optically anisotropic samples [5]. Since it is inexpensive and straightforward to align and calibrate, the MMP based on dual-rotating compensator layout still remains popular up to now [6]. By assuming that all the optical elements are perfect, the principles and the calibration method of the dual rotating-compensator MMP has been described [4]. However, this assumption is not true in practical application, as the finite bandwidth and the imperfect collimation would induce an apparent depolarization into the Mueller matrix of the compensator, and also the defective compensator itself is with some depolarization factors. These actual imperfect compensators, which have different Mueller matrix from the ideal ones, would lead to inaccurate calibration results and also accuracy loss in the final Mueller matrix measurement.

In this paper, we propose a method to calibrate the depolarization parameters of the compensators. The calibration method is a regression calibration method based on the Levenberg-Marquardt (LM) algorithm [7]. By using the iterative regression algorithm, the Mueller matrix dataset calculated from the response of the optical system are fitted to the exact expressions that model the response of the optical system. We also deduce a set of correction equations for the Mueller matrix calculation from the measured spectrum by taking into account the depolarization parameters of the compensators. To confirm the validity of the proposed correction method, we performed experiments on the air medium and a nominally 100nm thick SiO_2 thermal oxide film on a Si substrate with a home-made MMP, and expect to achieve higher measurement accuracy in the metrology.

2. Method

The assumed ideal compensator is represented by the Mueller matrix as [5]:

$$\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} M_{ideal} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\delta & \sin\delta \\ 0 & 0 & -\sin\delta & \cos\delta \end{bmatrix}, \tag{1}$$

where δ denotes the retardance of the compensator. However, in the actual dual rotating-compensator system, the finite bandwidth and imperfect collimation would induce an apparent depolarization into the Mueller matrix of the compensator, and also the defective compensator itself is with some depolarization factors. In this case, the actual Mueller matrix of the compensator is shown as:

$$M_{actual} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-c & 0 & 0 \\ 0 & 0 & (1-b)\cos\delta & (1-b)\sin\delta \\ 0 & 0 & -(1-b)\sin\delta & (1-b)\cos\delta \end{bmatrix}, \tag{2}$$

where b and c represent the depolarization parameters of the compensator.

In this case, the calibration system parameters of the dual rotating-compensator MMP involves the azimuthal angles of polarizer P, analyzer A, and the compensators Cs_1 and Cs_2, the retardance of the 1st and 2nd compensator δ_1 and δ_2 as well as the depolarization parameters of the 1st and 2nd compensator b_1, c_1 and b_2, c_2. When performing the regression calibration method, the Mueller matrix dataset would be used as the fitting data to extract the above system parameters. In actual practice, the measured Mueller matrix M_{ij}^{m} (i, j = 1, 2, 3, 4) can be calculated by the equations deduced in reference [4] with some correction factors described in Table 1. Thus, the measured Mueller matrix dataset M_{ij}^{m} can be represented as:

$$M_{ij}^{m} = f\left(P, A, Cs1, Cs2, \delta 1, \delta 2, b1, c1, b2, c2\right), \qquad (3)$$

while the predicted Mueller matrix dataset M_{ij}^{p} of the sample can be known as unit matrix for air and also can be calculated by the optical film transfer-matrix method for films [8]. Since the above system parameters are related to wavelength, the LM algorithm is employed here by minimizing the σ function versus wavelength to extract the system parameters. Here, the σ function represents the fitting error between the measured and predicted Mueller matrix elements by each wavelength defined as:

$$\sigma = \sum_{i,j=1}^{4} \left[M_{ij}^{m}\left(\lambda_k\right) - M_{ij}^{p}\left(\lambda_k\right) \right]^2, \qquad (4)$$

where λ_k denotes the kth spectral point. $M_{ij}^{m}\left(\lambda_k\right)$ is the measured data of the Mueller matrix elements with the kth wavelength, and $M_{ij}^{p}\left(\lambda_k\right)$ is the corresponding predicted value. After performing the proposed calibration method, the depolarization parameters of the compensators can be used to correct the Mueller matrix calculation by the correction equations shown in Table 1.

Table 1. The corrected Mueller matrix elements M_{ij}. Here m_{ij} is the corresponding Mueller matrix element calculated by the equations in reference [4].

$M_{11} = 1$	$M_{12} = m_{12}/(1-c_1)$	$M_{13} = m_{13}/(1-c_1)$	$M_{14} = m_{14}/(1-b_1)$
$M_{21} = m_{21}/(1-c_2)$	$M_{22} = m_{22}/[(1-c_1)*(1-c_2)]$	$M_{23} = m_{23}/[(1-c_1)*(1-c_2)]$	$M_{24} = m_{24}/[(1-b_1)*(1-c_2)]$
$M_{31} = m_{31}/(1-c_2)$	$M_{32} = m_{32}/[(1-c_1)*(1-c_2)]$	$M_{33} = m_{33}/[(1-c_1)*(1-c_2)]$	$M_{34} = m_{34}/[(1-b_1)*(1-c_2)]$
$M_{41} = m_{41}/(1-b_2)$	$M_{42} = m_{42}/[(1-c_1)*(1-b_2)]$	$M_{34} = m_{34}/[(1-c_1)*(1-b_2)]$	$M_{44} = m_{44}/[(1-b_1)*(1-b_2)]$

3. Results

All our experiments in this paper were performed on a home-made dual-rotating compensator MMP as shown in figure 1, where the light source is a deuterium (D2) and quartz tungsten halogen (QTH) combined source, the polarizer and the analyzer are the αBBO Rochon prisms, the compensators are the home-designed achromatic waveplates and the detector is a spectrograph which covering the spectrum from ultra-violet(UV) to Infra-red (IR).

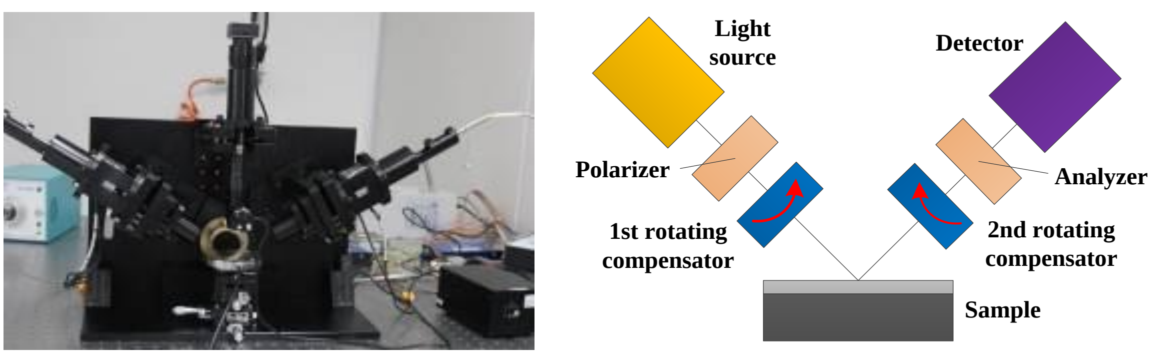

Fig. 1. Home-made dual rotating-compensator MMP.

To obtain the depolarization parameters of the compensators, we performed a measurement on the air medium by the MMP in the straight-through mode. The depolarization parameters of the 1st and 2nd compensator extracted from the proposed calibration procedure are shown in figure 2. To simplify the calculation, we assumed that the depolarization parameters of the two compensators are the same, b_1=b_2 and c_1=c_2. To verify the validity of the proposed correction method, another measurement on the air medium was performed and the obtained depolarization parameters were used as the correction terms to calculate the Mueller matrix of the air. A comparison of the Mueller matrix elements calculated from the same measured spectrum by the MMP with and without correction is shown in figure 3. It is clear that the measurement with the proposed correction method can achieve a much higher accuracy than the same measurement without any consideration of the imperfect compensators.

Therefore, we can conclude that the correction method proposed in this paper dramatically decreases the measurement error due to the effect of imperfect compensators and thus gives access to a higher accuracy in the actual Mueller matrix measurement.

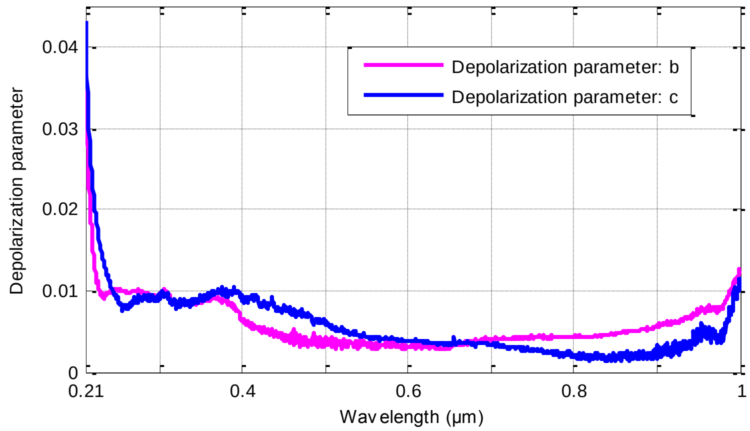

Fig. 2. Depolarization parameters of the compensators in the dual rotating-compensator MMP.

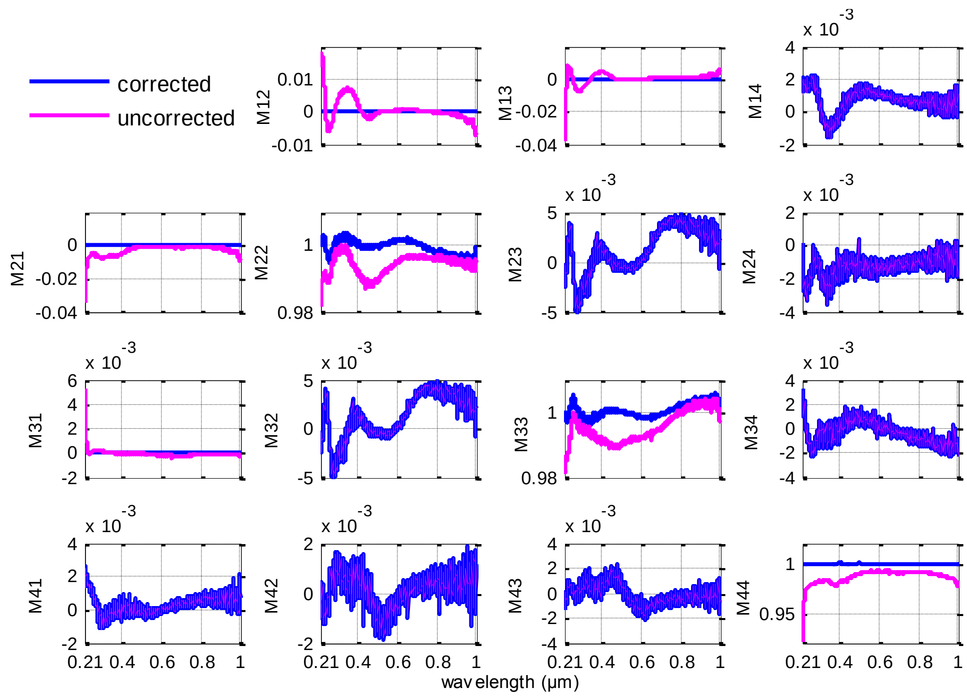

Fig. 3. Comparison between the Mueller matrix calculated from the same measured data by the MMP with and without the correction.

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