

# Image registration based on geometric pattern matching

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## ABSTRACT

We present an image registration method based on geometric pattern matching. The method is applicable to the situation that there is a similarity transformation between two images. First of all, the edges of the images were detected and the edge points were described with subpixel position and direction of gradient vector of intensities. The geometric denoising method was proposed to smooth the position and direction of the edge points. And then a coarse registration was performed to compute the similarity transformation parameters with pixel accuracy. In the end, a high accuracy registration based on iterative closest point (ICP) algorithm is used to obtain the final result.

**Keywords:** image registration, geometric pattern matching, edge, ICP

## 1. INTRODUCTION

Image registration [1, 2] has many applications, such as manufacturing industry, medicine, and military, and it is extensively studied in the past decades. The purpose of image registration is to look for the best transformation which precisely aligns two images by maximizing a similarity measure. There are lots of different image registration methods, which can be classified into two major categories: area-based and feature-based [2]. The method we described in this paper belongs to the latter one. The reason that the edges are used as feature for the registration is not only because edges are hardly affected by illumination changes but also because the computational costs can be reduced. In this paper, we present an accurate and robust image registration method for alignment system in integrated circuit (IC) fabrication equipments.

### 1.1. Related work

Classical area-based methods such as cross-correlation (CC) [3] employ image intensities for matching. As a result, they are sensitive to the changes of intensity, different illumination, shadow, and so on. The sequential similarity detection algorithm (SSDA) [4] is similar to CC, while it is computationally more efficient. The mutual information (MI) [5] is an important technique in multimodal image registration where images are obtained by different sensors. MI is extensively applied to medical image registration in recent years. The Fourier methods [6] transform the information of images from spatial domain to frequency domain and then use the phase correlation to facilitate image registration. All of the above-mentioned registration methods are excellent, yet defective in some aspect.

The other category of image registration method is the feature-based method. Points, lines and regions can be considered as the features. The point features can be corners, intersection point of lines, centroid of region, and so on. The line features include edges, contours, line segments, and so on. The region features are closed-boundary regions. Rucklidge [7] used Hausdorff distance between two point sets to look for an affine transformation of a model in an image, but it is difficult to locate the target with subpixel accuracy. The generalized Hough transformation proposed by Ballard [8] is another image registration method, while the computational cost is high. The geometric hashing method [9] can perform the matching efficiently, if the number of geometric primitives is not so large. Hsieh et al. [10] proposed an edge-based approach for image registration, which used the angle histogram, but only about 10% scaling variation can be tolerated. Lowe [11] presented a scale invariant feature named scale invariant feature transform (SIFT) to matching objects. Matas et al. [12] developed the maximally stable external region (MSER), and the MSER feature can be used for image registration.

## 1.2. Approach and contribution

We describe an image registration method based on geometric pattern matching for alignment system in IC fabrication equipments. Given two images (one is template image, and the other is target image), it locates the template image in the target image with subpixel accuracy. Our approach consists of three major steps. Firstly, it detects edges in the two images with subpixel accuracy and describes the edge points by position and direction. Any variable associated with template image edge points is denoted with letter  $p$  and target image edge points by  $q$  in this paper. Secondly, we create a template using the edge points of the template image and then employ a coarse-to-fine hierarchical search strategy to carry out coarse registration, which accelerates the registration observably. Finally, taking the result of coarse registration as an initial transformation pose, we use the well-known iterative closest point (ICP) algorithm [13] to finish the high accuracy image registration.

The main contribution in the paper is that a geometric de-noising method is proposed to make the edges insensitive to noise. To remove the noise in real images, smoothing filters such as the Gaussian filter are often used. However, the conventional de-noising methods are suitable for smoothing the intensity of image pixels but not for smoothing geometric features, such as position and direction of edge points. In these cases, it is difficult for the conventional intensity based filters to achieve robustness and high accuracy. For this reason, we use linear fitting to locally fit the subpixel coordinates of edge points to a straight line segment, thus smoothing the coordinates and normal direction of the edge points. We name the technique geometric de-noising method.

This paper is organized as follows. In section 2 the theory and methodology are presented completely. Section 3 shows experiments and results in detail. In section 4, we conclude our work and discuss the future possibilities.

## 2. THEORY AND METHODOLOGY

### 2.1. Edge detection and feature description

A digital image is often defined as a two-dimensional discrete function  $g(x, y)$ , where  $x$  and  $y$  are spatial coordinates, i.e. position, and  $g$  is the intensity. Edges are discontinuities of the intensity in an image, and many edge detection methods are based on the derivatives of intensity function  $g$ . In this paper, the Canny edge detector is employed to obtain the position of edge points with pixel accuracy and the gradient direction of edge points [14]. Let  $g_x$  be partial derivative in  $x$  axis and  $g_y$  in  $y$  axis, the direction of gradient vector is computed by  $\arctan(g_y / g_x)$ . However, edge points with pixel accuracy cannot result in subpixel accuracy image registration. Therefore, an edge detection method that has sub-pixel accuracy is needed. In our present study, a simple sub-pixel edge detection method is used [15]. In this method, for each edge point, one uses the norm of gradient vector of the edge point and those of its two adjacent points in its gradient direction to interpolate a quadratic function, and the position of the extreme is taken as the subpixel position of the edge point [15].

Although the subpixel edge detection method mentioned above is simple, it is sensitive to noise, thus usually delivering unsmooth edges. In order to remove noises along the edges, the disordered edge points are first connected into chains, then short chains are deleted, and finally geometric de-noising method is applied to each chain to filter the position and direction of the edge points. We treat the template image edge points on a chain as a point set of  $n$  points  $P = \{p_1, p_2, \dots, p_n\}$ ,  $p_i \in \mathbb{R}^2$ , and the subpixel position of edge point  $p_i$  is denoted as  $p_i = (x_{p_i}, y_{p_i})$ . We use  $m$  points to fit a line segment, where  $m$  is an odd integer (e.g., 5, 7 or 9). The  $k$ -th point is the center of  $m$  points, i.e.  $k = (m + 1) / 2$ , and its position and direction are determined by the fitted line (see Figure 1). The line equation is written as

$$\rho = x \cos \theta + y \sin \theta \quad (1)$$

where  $\rho$  is the distance between the origin and the line, and  $\theta$  is the angle from  $x$  axis to  $\rho$  counterclockwise.

We use the least-squares method to calculate the parameters of the fitted line. Let  $e$  be half of the sum of the square of the distance between each point  $p_i$  ( $i = 1, 2, \dots, m$ ) and the fitted line

$$e = \frac{1}{2} \sum_{i=1}^m (x_{p_i} \cos \theta + y_{p_i} \sin \theta - \rho)^2 \quad (2)$$

The minimum of  $e$  is achieved where the first-order partial derivatives are zero, that is

$$\begin{cases} \frac{\partial e}{\partial \rho} = \sum_{i=1}^m (x_{pi} \cos \theta + y_{pi} \sin \theta - \rho)(-1) = 0 \\ \frac{\partial e}{\partial \theta} = \sum_{i=1}^m (x_{pi} \cos \theta + y_{pi} \sin \theta - \rho)(-x_{pi} \sin \theta + y_{pi} \cos \theta) = 0 \end{cases} \quad (3)$$

The solution of equations (3) gives two parameters  $\rho$  and  $\theta$  of the fitted line.

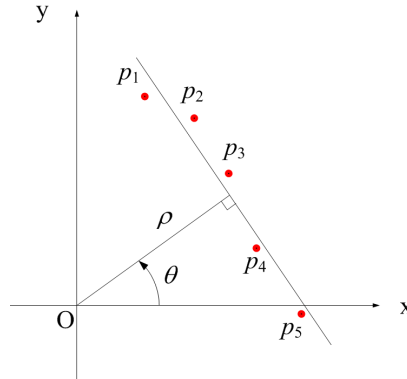


Figure 1. Five points are used to fit a line and the 3rd point is the point in question.

We project the original  $k$ -th point onto the fitted line and the position of the projection is considered as the new position of the  $k$ -th point. The direction of the  $k$ -th point is corrected at the same time. The difference between the normal direction of the fitted line and the gradient direction of the  $k$ -th point is compared. If the absolute value of the difference is less than a threshold, e.g. 45 degree, the new gradient direction of the  $k$ -th point is set to the normal direction of the fitted line, otherwise it is set to the opposite direction. The projected position of the  $k$ -th point  $(x'_{pk}, y'_{pk})$  is given by

$$\begin{cases} x'_{pk} = \rho \cos \theta + x_{pk} \sin^2 \theta - y_{pk} \sin \theta \cos \theta \\ y'_{pk} = \rho \sin \theta + y_{pk} \cos^2 \theta - x_{pk} \sin \theta \cos \theta \end{cases} \quad (4)$$

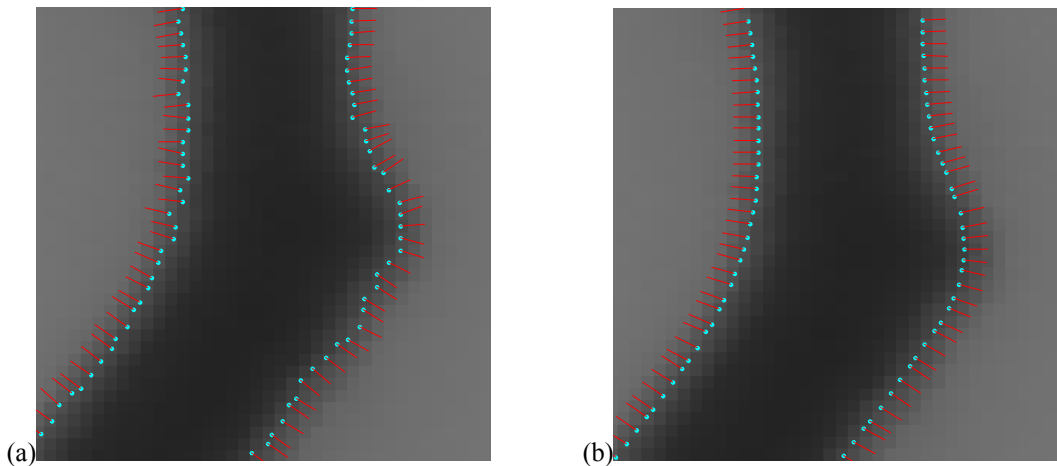


Figure 2. Comparison of edge points before and after geometric de-noising. (a) subpixel position and direction of edge points before geometric de-noising. (b) subpixel position and direction of the edge points after geometric de-noising.

After detecting subpixel edges in the image and de-noising the position and direction of the edge points, the feature that consists of the subpixel position and direction of the edge points is more accurate and robust, as shown in Figure 2.

## 2.2. Coarse registration

A similarity measure is used for the coarse registration. This similarity measure uses the gradient direction of edge points, and it is proportional to the direction coherence between the template image edge points and the target image edge points. If all point pairs have the same direction, the similarity measure will be 1, which means completely matching. The similarity measure [16, 17] is defined by

$$sim = \frac{1}{n} \sum_{i=1}^n \frac{\langle \mathbf{g}'_{pi}, \mathbf{g}_{qi} \rangle}{\|\mathbf{g}'_{pi}\| \|\mathbf{g}_{qi}\|} = \frac{1}{n} \sum_{i=1}^n \frac{(\mathbf{g}'_{pix} \mathbf{g}_{qix} + \mathbf{g}'_{piy} \mathbf{g}_{qiy})}{\sqrt{\mathbf{g}'_{pix}{}^2 + \mathbf{g}'_{piy}{}^2} \sqrt{\mathbf{g}_{qix}{}^2 + \mathbf{g}_{qiy}{}^2}} \quad (5)$$

where  $\mathbf{g}'_{pi} = (\mathbf{g}'_{pix}, \mathbf{g}'_{piy})$  is the gradient of the template edge point  $p_i$ .  $\mathbf{g}_{qi} = (\mathbf{g}_{qix}, \mathbf{g}_{qiy})$  is the gradient of the target edge point  $q_i$ .  $\langle \mathbf{g}'_{pi}, \mathbf{g}_{qi} \rangle$  is the inner product of  $\mathbf{g}'_{pi}$  and  $\mathbf{g}_{qi}$ .  $\|\cdot\|$  is the L2 norm.

A template is created by using the edges extracted from the template image before carrying out image registration. The template has a pyramid structure [17]. The number of levels of the pyramid should be chosen carefully so that the pattern at the top level is discernible. The pyramid for the target image has the same number of levels as the template image.

After the construction of pyramids, a complete searching is performed by using the similarity measure at the top level of pyramid [17]. Because of the small size of the images at the top level, the computational cost is very small. The results obtained at the top level are then used as an initial guess for the searching at the next level [17]. Except at the top level, the searching is performed only in a small window around the results obtained at the higher level. Since the search space is reduced, the coarse registration is efficient.

The results of coarse registration are of pixel accuracy. Four parameters of similarity transformation, i.e., scale parameter  $s$ , rotation parameter  $\alpha$ , translation parameters  $t_x$  and  $t_y$ , between two images are obtained, and they are treated as the initial guess for the ICP algorithm to calculate more accurate results.

## 2.3. ICP with scale changing

The template image edge points can be taken as a point set  $P$ , and the target image edge points a point set  $Q$ . Denote 4 parameters of the results of coarse registration as  $s_0$ ,  $\alpha_0$ ,  $t_{x0}$  and  $t_{y0}$ . The ICP algorithm is used in our present work to achieve high accuracy image registration. The main process of ICP [13, 18] is described as follows:

- i. Input two point sets  $P_0 = \{p_1, p_2, \dots, p_n\}$  and  $Q = \{q_1, q_2, \dots, q_m\}$ , scale parameter  $s_0$ , rotation parameter  $\alpha_0$ , translation parameters  $t_{x0}$  and  $t_{y0}$ .
- ii. Find corresponding points. If the distance between two closest points  $p_i$  and  $q_j$  is less than a threshold and the difference between the direction of  $p_i$  and  $q_j$  is less than a threshold,  $p_i$  and  $q_j$  are considered as a pair of corresponding points.
- iii. Compute registration parameters. We minimize the sum of the distance between template points to the line that passes through the corresponding target point and perpendicular to the normal direction of the target point, and thus we obtain  $s_{k+1}$ ,  $\alpha_{k+1}$ ,  $t_{xk+1}$  and  $t_{yk+1}$ .
- iv. Compute error. Apply the registration to  $P_k$  to obtain  $P_{k+1}$ , and calculate the difference between the point set  $P_{k+1}$  and the point set  $Q$ .
- v. If  $k$  reaches the maximum number of iterations or the change in error falls below a threshold, terminate the iteration. Otherwise go to step iii.
- vi. Output the scale parameter  $s$ , rotation parameter  $\alpha$ , translation parameters  $t_x$  and  $t_y$ .

The computational load of ICP is proportional to the number of points. Down-sampling of the feature can be employed to accelerate ICP.

## 3. EXPERIMENTS AND RESULTS

### 3.1. Experiments on real images

We tested the registration method with real world images that are obtained in the process of IC fabrication. An example is shown in Figure 3. The left image is the template image, and the right image is the target image with the template image located. The size of the template image and the target image are  $600 \times 600$  pixels and  $1392 \times 1040$  pixels respectively. We can see that the template image is located in the target image correctly.

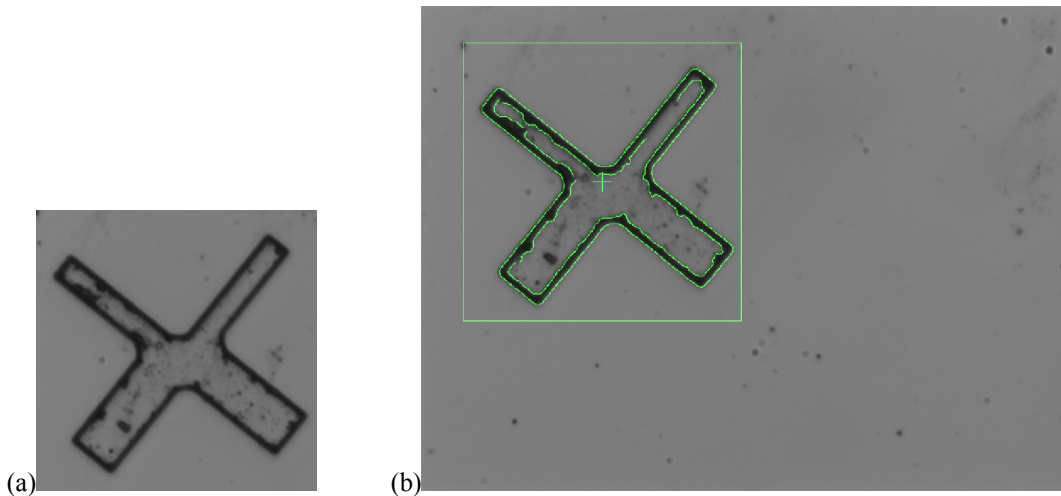


Figure 3. Real world images obtained in IC fabrication. (a) template image (size 600×600 pixels). (b) target image (size 1392×1040 pixels) and the template image is located correctly.

### 3.2. Comparison with PatMax

VisionPro is a computer vision software of Cognex Corporation, and PatMax is a tool of the software for high accurate registration. Therefore, we compared our results with those of PatMax. Figure 4 shows the error of registration of the first group of images, where the size of the template image is 600×600 pixels and the target image 1392×1040 pixels. Similarly, Figure 5 shows the error of the second group of images, where the size of the template image is 420×420 pixels and the target image 1392×1040 pixels. Figure 6 shows the error of the third group of images, where the size of the template image is 600×600 pixels and the target image 1392×1040 pixels. As Figure 4-6 show, the maximal deviation of error is around 0.1 pixel.

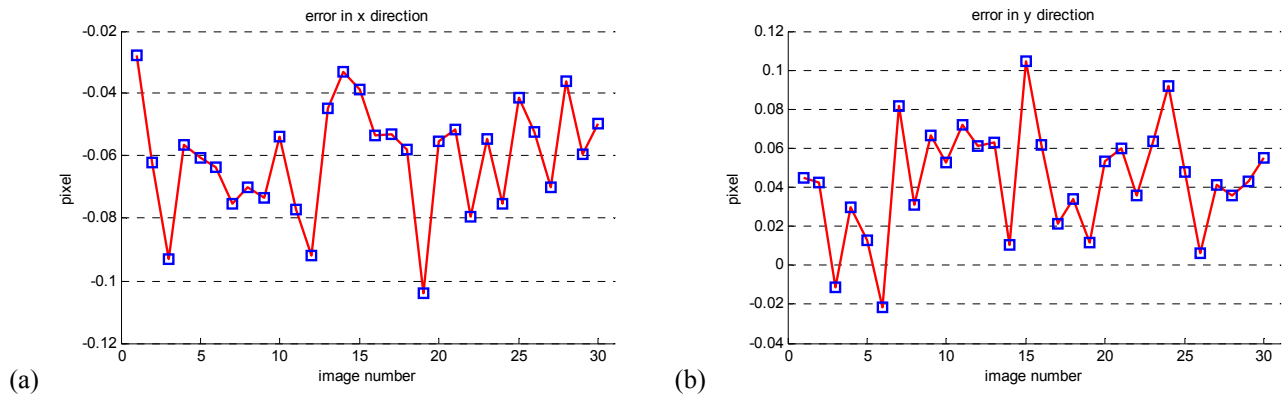


Figure 4. Results of registration of the first group of images. (a) translation error in x direction. (b) translation error in y direction.

For the purpose of measuring the robustness and accuracy quantitatively, we calculated the maximum, minimum, mean, standard deviation of the registration error and recorded them in Table 1. The maximum of error is 0.1049 pixels and the minimum of error is  $-0.104$  pixels. The worst mean of error is  $-0.0618$  pixels and the worst standard deviation of error is 0.0278 pixels. The error of our results can be considered as the data following the normal distribution. As we know, 99.7 percent of the data values are within three standard deviations  $\sigma$  of the mean (mathematically,  $\text{mean} \pm 3\sigma$ ). The values of  $\text{mean} \pm 3\sigma$  of the three groups are: first group  $-0.1131$  pixels and  $-0.0104$  pixels in x direction,  $-0.0407$  pixels and 0.1263 pixels in y direction; second group  $-0.1102$  pixels and 0.0168 pixels in x direction,  $-0.025$  pixels and 0.1114 pixels in y direction; third group  $-0.0197$  pixels and 0.0704 pixels in x direction,  $-0.0425$  pixels and 0.0813 pixels in y direction. The statistical data show that the accuracy of our method is higher than 0.15 pixels.

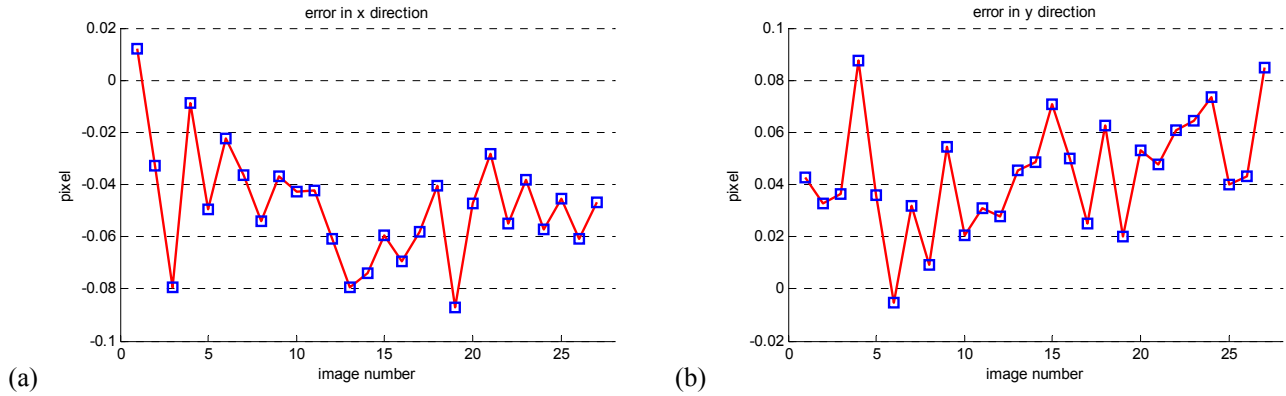


Figure 5. Results of registration of the second group of images. (a) translation error in x direction. (b) translation error in y direction.

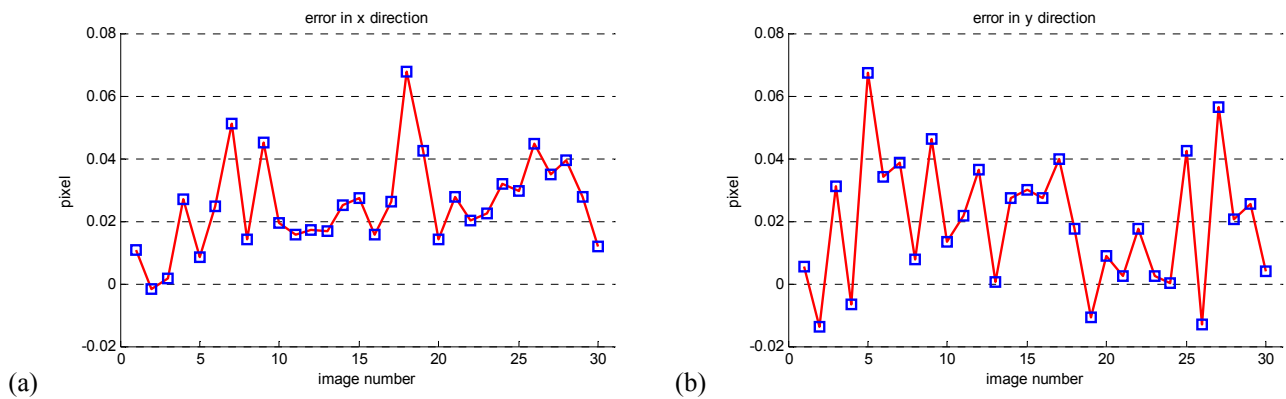


Figure 6. Results of registration of the third group of images. (a) translation error in x direction. (b) translation error in y direction.

Table 1. Statistical data (in units of pixels) of registration error.

Testing images	Maximum		Minimum		Mean		Standard deviation	
	x	y	x	y	x	y	x	y
First group	-0.0278	0.1049	-0.1040	-0.0213	-0.0618	0.0428	0.0171	0.0278
Second group	0.0121	0.0875	-0.0870	-0.0053	-0.0467	0.0432	0.0212	0.0227
Third group	0.0679	0.0673	-0.0018	-0.0138	0.0253	0.0194	0.0150	0.0206

#### 4. CONCLUSIONS

In the paper an image registration method based on geometric pattern matching is presented. We use the proposed geometric de-noising method to smooth the position and direction of the edge points. A well-performed similarity measure is used for template searching. A coarse-to-fine strategy is employed to accelerate the image registration. We make use of the pyramid construction to reduce the computational load greatly. In order to reach the high accuracy, the well-known ICP algorithm is performed after coarse registration. The experiments demonstrate that the method is robust and accurate. The registration results of the real world images obtained in IC fabrication are better than 0.15 pixels in x and y direction.

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