Fast Evaluation of Aberration-Induced Intensity Distribution in Partially Coherent Imaging Systems by Cross Triple Correlation *

LIU Shi-Yuan(刘世元)[1,2**], LIU Wei(刘巍)[1], WU Xiao-Fei(吴小飞)[2]

[1] *Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074*

[2] *State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074*

(Received 30 March 2011)

We propose a method suitable for the fast calculation and evaluation of aberration-induced intensity distribution in partially coherent imaging systems, such as projection lithographic tools. The method is based on transmission cross coefficient (TCC) decomposition by a general operator, namely cross triple correlation (CTC). By expanding the aberrated pupil function into a Taylor series, the TCC is decomposed into different terms. Each term is further represented as a weighted sum of several CTCs. By exploring the properties of CTC, the aerial image intensity induced by wavefront aberration is calculated quickly and separated clearly from that without aberration. Simulation results and discussion are presented.

PACS: 42.30.Va, 42.15.Fr, 42.30.Lr, 85.40.Hp DOI:10.1088/0256-307X/28/10/104212

With ever decreasing feature sizes, the impact of lens aberration has become increasingly important for imaging quality control of projection lithographic tools.[1−4] The imaging optics configuration in lithographic tools is typically a partially coherent system characterized by the intensity distribution of the source and the pupil function of the projection lens. Imaging properties of such partially coherent systems have to be described using a bilinear model,[5] which leads to time-consuming calculations and understanding difficulties,[6,7] especially in cases where wavefront aberrations are involved. Recently we proposed a fast algorithm for the quadratic aberration model in optical lithography.[8] Although this approach adopts a mathematical operator to achieve the fast calculation of aberration-induced intensity distributions, this approach is not suitable for optical lithography simulations under a relatively large amount of arbitrary input aberrations, as the quadratic model is a simplified imaging model on the assumption of a small amount of aberrations in the projection lens. Therefore, it is highly desirable to develop a more generalized method suitable for fast calculation, clear separation and efficient evaluation of aberration-induced intensity distributions in partially coherent imaging systems.[9]

In this Letter, we propose a method with rigorous formulations that can be suitable for the fast evaluation of aberration-induced intensity distribution in partially coherent imaging systems. The proposed method is especially suitable for arbitrary input aberrations and is based on a general operator, namely *cross triple correlation* (CTC), which is defined as[10]

$$C_{KLM}(\boldsymbol{f}_1,\boldsymbol{f}_2)=\int K(\boldsymbol{f})L(\boldsymbol{f}+\boldsymbol{f}_1)M(\boldsymbol{f}+\boldsymbol{f}_2)d\boldsymbol{f},\tag{1}$$

where $K(\boldsymbol{f})$, $L(\boldsymbol{f})$, and $M(\boldsymbol{f})$ are three different functions. Variables $\boldsymbol{f}$, $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ can be any real scalars for a one-dimensional signal, but through this study they are two-dimensional real vectors representing the normalized spatial-frequency pupil coordinates. Most of the physical functions required for the analysis of lithographic systems have a compact support and the integration can be treated over the whole space of $\boldsymbol{f}$.

A well-established bilinear model for partially coherent systems is based on the Hopkins theory with the concept of the transmission cross coefficient (TCC), which is expressed as[11]

$$T(\boldsymbol{f}_1,\boldsymbol{f}_2)=\int S(\boldsymbol{f})H(\boldsymbol{f}+\boldsymbol{f}_1)H^*(\boldsymbol{f}+\boldsymbol{f}_2)d\boldsymbol{f},\tag{2}$$

where $S(\boldsymbol{f})$ is the effective source function and $H(\mathbf{f})$ is the pupil function given by

$$H(\boldsymbol{f})=P(\boldsymbol{f})\exp[-ikW(\boldsymbol{f})].\tag{3}$$

Here $P(\boldsymbol{f})=\text{circ}(\boldsymbol{f})$ is the unaberrated pupil function, $k=2\pi/\lambda$ is the wave number; λ is the wavelength of the monochromatic light source, and $W(\boldsymbol{f})$ is the aberrated wavefront including the lens aberration and the defocus aberration.[4] The image intensity at a space point $\boldsymbol{x}$ is then written in terms of the pairs of spatial-frequencies of the object spectra $O(\boldsymbol{f}_1)$ and $O(\boldsymbol{f}_2)$:[11]

$$I(\boldsymbol{x})=\iint O(\boldsymbol{f}_1)O^*(\boldsymbol{f}_2)T(\boldsymbol{f}_1,\boldsymbol{f}_2)\cdot\exp[-2\pi i(\boldsymbol{f}_1-\boldsymbol{f}_2)\cdot\boldsymbol{x}]d\boldsymbol{f}_1d\boldsymbol{f}_2.\tag{4}$$

It is interesting to note that the expression of the TCC in Eq. (2) is quite similar to the general operator CTC in Eq. (1), with the difference that the last two functions involved in the TCC are two conjugate pupils instead of the two fully independent functions in CTC. Therefore, the TCC can be considered a special case of CTC, which involves shifting the pupil function and its conjugation, and then multiplying by the source

*Supported by the National Natural Science Foundation of China under Grant No 91023032, and Fundamental Research Funds for the Central Universities of China under Grant No 2010ZD004.

**Correspondence author. Email: shyliu@mail.hust.edu.cn

function. Substituting Eq. (3) into Eq. (2) and noticing that $P(\boldsymbol{f}) = P^*(\boldsymbol{f})$, the expression for TCC in the case that a wavefront aberration $W(\boldsymbol{f})$ is induced can be written as

$$T(\boldsymbol{f}_1, \boldsymbol{f}_2) = \int S(\boldsymbol{f})P(\boldsymbol{f}+\boldsymbol{f}_1)P(\boldsymbol{f}+\boldsymbol{f}_2) \cdot \exp\{-ik[W(\boldsymbol{f}+\boldsymbol{f}_1) - W(\boldsymbol{f}+\boldsymbol{f}_2)]\}d\boldsymbol{f}. \tag{5}$$

Note that the exponential function inside the integral can be represented as an infinite Taylor series:

$$\exp\{-ik[W(\boldsymbol{f}+\boldsymbol{f}_1) - W(\boldsymbol{f}+\boldsymbol{f}_2)]\} = \sum_{n=0}^{\infty} \frac{(-ik)^n}{n!}[W(\boldsymbol{f}+\boldsymbol{f}_1) - W(\boldsymbol{f}+\boldsymbol{f}_2)]^n, \tag{6}$$

and each term of the Taylor series can be further expanded by applying the binomial theorem:

$$[W(\boldsymbol{f}+\boldsymbol{f}_1) - W(\boldsymbol{f}+\boldsymbol{f}_2)]^n = \sum_{m=0}^{n} \frac{(-1)^{n-m}n!}{m!(n-m)!}[W(\boldsymbol{f}+\boldsymbol{f}_1)]^m[W(\boldsymbol{f}+\boldsymbol{f}_2)]^{n-m}. \tag{7}$$

Therefore, the TCC in Eq. (5) can be decomposed into different terms and each term can be further represented as a weighted sum of several CTCs:

$$T(\boldsymbol{f}_1, \boldsymbol{f}_2) = \sum_{n=0}^{\infty} T_n(\boldsymbol{f}_1, \boldsymbol{f}_2), \tag{8}$$

$$T_n(\boldsymbol{f}_1, \boldsymbol{f}_2) = (ik)^n \sum_{m=0}^{n} \frac{(-1)^m}{m!(n-m)!} C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2), \tag{9}$$

where $C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2)$ is a special CTC with the following notation:

$$C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2) = \int S(\boldsymbol{f})\{P(\boldsymbol{f}+\boldsymbol{f}_1)[W(\boldsymbol{f}+\boldsymbol{f}_1)]^m\} \cdot \{P(\boldsymbol{f}+\boldsymbol{f}_2)[W(\boldsymbol{f}+\boldsymbol{f}_2)]^{n-m}\}d\boldsymbol{f}. \tag{10}$$

From these definitions, it is obvious that the first term with $n = 0$ in Eq. (8) represents the TCC without aberration, thus we name it the unaberrated TCC. All the other terms in Eq. (8) from $n = 1$ to ∞ display the influence of aberrations. We call them the linearly aberrated TCC when $n = 1$, the quadratically aberrated TCC when $n = 2, \ldots,$. Consequently, the total image intensity can be decomposed and represented in the following formulations:

$$I(\boldsymbol{x}) = \sum_{n=0}^{\infty} I_n(\boldsymbol{x}), \tag{11}$$

$$I_n(\boldsymbol{x}) = (ik)^n \sum_{m=0}^{n} \frac{(-1)^m}{m!(n-m)!} J_{n;m,n-m}(\boldsymbol{x}), \tag{12}$$

$$J_{n;m,n-m}(\boldsymbol{x}) = \iint O(\boldsymbol{f}_1)O^*(\boldsymbol{f}_2)C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2) \cdot \exp[-2\pi i(\boldsymbol{f}_1 - \boldsymbol{f}_2)\cdot \boldsymbol{x}]d\boldsymbol{f}_1 d\boldsymbol{f}_2, \tag{13}$$

where $I_0(\boldsymbol{x})$ is called aberration-free intensity and $I_n(\boldsymbol{x})$ displays the aberration-induced intensity distribution of the nth order (n from 1 to ∞). For instance, $I_1(\boldsymbol{x})$, $I_2(\boldsymbol{x})$ and $I_3(\boldsymbol{x})$ indicate the linear, quadratic and cubic terms of the aberration-induced intensity, respectively. $J_{n;m,n-m}(\boldsymbol{x})$ is the basic intensity component which can be treated as a kernel for calculating the corresponding aberration-induced intensity $I_n(\boldsymbol{x})$.

Although auto triple correlation (ATC), or briefly triple correlation (TC), in which the correlated three functions in Eq. (1) is the same, has been intensively investigated and widely used in digital and optical signal processing,[10] CTC is much less explored. Here we list some of the useful properties of the general CTC defined in Eq. (1):

$$\hat{C}_{KLM}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \hat{K}(\boldsymbol{x}_1)\hat{L}(\boldsymbol{x}_2)\hat{M}(-\boldsymbol{x}_1 - \boldsymbol{x}_2), \tag{14}$$

$$C_{KML}(\boldsymbol{f}_2, \boldsymbol{f}_1) = C_{KLM}(\boldsymbol{f}_1, \boldsymbol{f}_2). \tag{15}$$

The property in Eq. (14) where $\hat{C}_{KLM}$, $\hat{K}$, $\hat{L}$ and $\hat{M}$ respectively represent the Fourier transformation of C_{KLM}, K, L and M, can directly lead to fast algorithms for the CTC calculation as the time-consuming integration in Eq. (1) is avoided and replaced by the simple multiplication of $\hat{K}$, $\hat{L}$ and $\hat{M}$. The property in Eq. (15) shows the condition for function and variable commutation, indicating that $C_{KLM}(\boldsymbol{f}_2, \boldsymbol{f}_1) \neq C_{KLM}(\boldsymbol{f}_1, \boldsymbol{f}_2)$ generally holds in the case $L \neq M$. If we let $A(\boldsymbol{f}_1, \boldsymbol{f}_2) = C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2) + C_{n;n-m,m}(\boldsymbol{f}_1, \boldsymbol{f}_2)$ and $B(\boldsymbol{f}_1, \boldsymbol{f}_2) = i[C_{n;m,n-m}(\boldsymbol{f}_1, \boldsymbol{f}_2) + C_{n;n-m,m}(\boldsymbol{f}_1, \boldsymbol{f}_2)]$, it can be derived from Eq. (10) that both $A(\boldsymbol{f}_1, \boldsymbol{f}_2)$ and $B(\boldsymbol{f}_1, \boldsymbol{f}_2)$ are Hermitian symmetric, i.e., $A(\boldsymbol{f}_1, \boldsymbol{f}_2) = A^*(\boldsymbol{f}_2, \boldsymbol{f}_1)$ and $B(\boldsymbol{f}_1, \boldsymbol{f}_2) = B^*(\boldsymbol{f}_2, \boldsymbol{f}_1)$. Applying this property and noticing the notations in Eq. (13), it is observed that the calculation results of $[J_{n;m,n-m}(\boldsymbol{x}) + J_{n;n-m,m}(\boldsymbol{x})]$ and $i[J_{n;m,n-m}(\boldsymbol{x}) - J_{n;n-m,m}(\boldsymbol{x})]$ both are pure real values. Thus, the real and imaginary parts of the basic intensity components $J_{n;m,n-m}(\boldsymbol{x})$ and $J_{n;n-m,m}(\boldsymbol{x})$ satisfy the relationship

$$\begin{aligned} &Re[J_{n;m,n-m}(\boldsymbol{x})] = \mathrm{Re}[J_{n;n-m,m}(\boldsymbol{x})], \\ &Im[J_{n;m,n-m}(\boldsymbol{x})] = -\mathrm{Im}[J_{n;n-m,m}(\boldsymbol{x})]. \end{aligned} \tag{16}$$

Substituting Eq. (16) into Eq. (12), the aberration-induced intensity distribution can be written as

$$I_n(\boldsymbol{x}) = \begin{cases} (ik)^n \sum\limits_{m=0}^{n/2} \frac{2\cdot(-1)^m}{m!(n-m)!}\mathrm{Re}[J_{n;m,n-m}(\boldsymbol{x})], \\ \quad \text{when } n = \text{even}, \\ i^{n+1}k^n \sum\limits_{m=0}^{(n-1)/2} \frac{2\cdot(-1)^m}{m!(n-m)!}\mathrm{Im}[J_{n;m,n-m}(\boldsymbol{x})], \\ \quad \text{when } n = \text{odd}, \end{cases} \tag{17}$$

From Eq. (17), it is observed that all the intensities $I_n(\boldsymbol{x})$ (n from 0 to ∞) are positive real values which successfully accord with the natural behavior of the light intensity propagation. It is also noted that the newly developed formulations are a rigorous physical model which is generalized and suitable for arbitrary input aberrations. In fact, if we consider a small amount of aberrations in the projection lens, Eq. (17) will be simplified to the quadratic aberration model reported by Ref. [8].

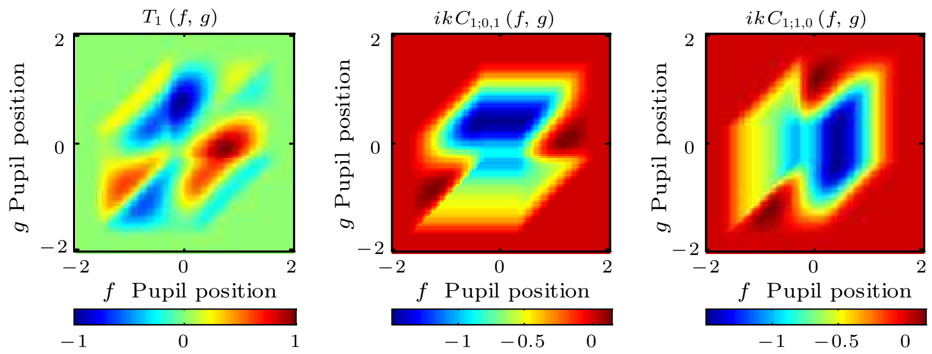

Fig. 1. Representation of the linear aberrated TCC term $T_1(\boldsymbol{f}_1, \boldsymbol{f}_2)$ as a sum of weighted CTCs.

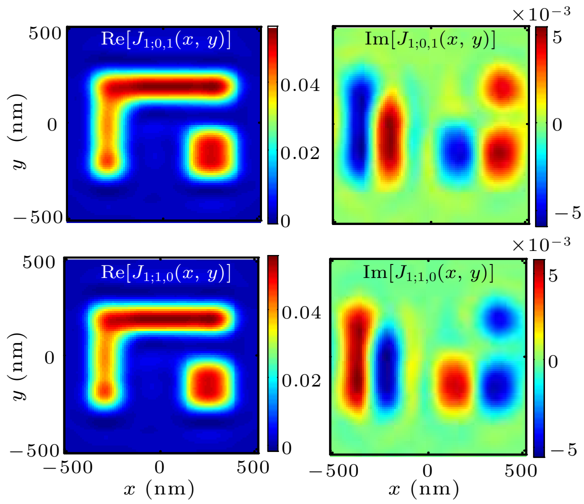

Fig. 2. The relationship of the basic intensity components.

Figure 1 depicts an example of calculating CTCs for the linearly aberrated TCC term $T_1(\boldsymbol{f}_1, \boldsymbol{f}_2)$. The wavelength used in simulation is 193 nm with a fixed numerical aperture (NA) of 0.75 and a defocus of 100 nm. The source S is set to be a circular one with a partial coherence of 0.7. An aberrated wavefront shown in Fig. 3 is induced by setting the Zernike coefficients $Z_5 = 0.1\lambda$ and $Z_7 = 0.1\lambda$, where the Z_5 and Z_7 indicate even type and odd type aberrations, respectively.[12] Since each CTC and TCC are both represented by four-dimensional matrices in Cartesian coordinates $(f,\, g;\, f',\, g')$, the cross-sections of the corresponding CTC and TCC can be depicted by two-dimensional matrices in Cartesian coordinates (f,g) as shown in Fig. 1. Although all the CTCs themselves in Eq. (10) are real values, the linear term $T_1(f,g)$ in Eq. (8) is purely imaginary. This is due to the fact that the aberration $W(\boldsymbol{f})$ itself is real but its exponential function term for the phase change is complex and the linear term in the expansion is imaginary. In addition, the $T_1(f,g)$ shown in Fig. 1 is obtained by a weighted sum of the CTCs in Eq. (9), as the expression of $T_1(f,g) = ik[C_{1;0,1}(f,g) - C_{1;1,0}(f,g)]$. It is also interesting to note that pairs of CTCs, such as $C_{1;0,1}(f,g)$ and $C_{1;1,0}(f,g)$ shown in Fig. 1, are exactly the same but the coordinates (f,g) are exchanged with each other. This is due to the commutation property as expected from Eq. (15).

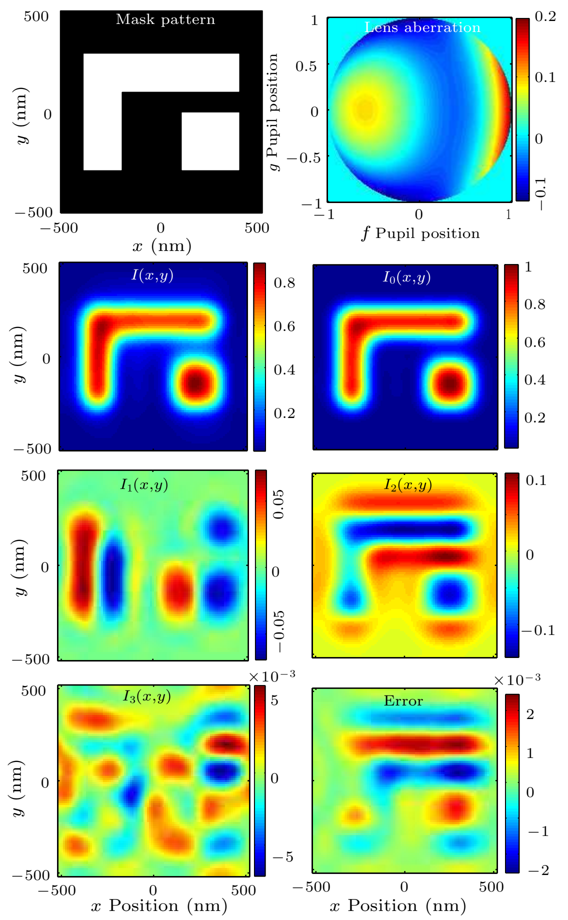

Fig. 3. Fast evaluation of aberration-induced intensity distributions by using the CTC-based method.

Calculation results of the basic intensity components $J_{1;0,1}(x,y)$ and $J_{1;1,0}(x,y)$ are shown in Fig. 2, based on the input mask shown in Fig. 3. Other simulation settings are the same as in Fig. 1. The simulated basic intensity components $J_{1;0,1}(x,y)$ and $J_{1;1,0}(x,y)$ are both complex values with real and imaginary parts, and their relationship demonstrates the derivation of Eq. (16). Therefore, for the purpose of aerial image calculation, we need to calculate only half of the CTCs in Eq. (9). Figure 3 depicts the aerial image calculation result, in which the aberration-induced intensity distributions for different terms are clearly evaluated and separated from each other. It is also noted that the linear term $I_1(x,y)$ and the quadratic term $I_2(x,y)$ play a more significant role than the higher terms such as the cubic term in the description of aberration-induced intensity distributions, with an error $e(x,y) = I(x,y) - \sum_{n=0}^{2} I_n(x,y)$ on the order of 10^{-3}. Therefore, it is usually enough to only calculate the linear term $I_1(x,y)$ and the quadratic term $I_2(x,y)$ for the fast evaluation of aberration-induced intensity distributions, but the cubic or higher terms should be taken into account if higher accuracy is needed.

The above simulations validate the proposed method and especially confirm the symmetric properties between the basic intensity components shown in Eq. (16). If we exploit some symmetric properties commonly found in the real-world lithographic tools,[13] we can further derive simplified formulations for the CTC and intensity calculations. If we adopt analytical forms of commonly used illumination sources such as annular, dipole and quadrupole, and express the wavefront aberration in Zernike representation, we can develop fully analytical algorithms for the CTC and intensity calculations with quite high efficiency and accuracy. Furthermore, algorithms based on the optimal coherent approximation (OCA) of TCC and currently used in the full-chip optical proximity correction (OPC) and inverse mask design,[14] can also be adopted and extended for the CTC. Since the individual CTCs and the CTC-based intensities can be summed up as the total TCC and intensity, respectively, they are quite suitable for parallel computations.

In summary, we propose a CTC-based method suitable for fast calculation and evaluation of aberration-induced intensity distribution in partially coherent imaging systems. By decomposition of the TCC into CTCs and by exploiting the properties of CTC, the aberration-induced intensities can be calculated quickly and separated clearly from each other. It is expected that this method will have applications in the robust OPC and inverse mask design with aberrations taking into account. It will also have applications in aerial image based aberration analysis and metrology.

References

[1] Smith B W and Schlief R 2000 *Proc. SPIE* **4000** 294
[2] Yuan Q Y, Wang X Z, Qiu Z C, Wang F, Ma M Y and He L 2007 *Opt. Express* **15** 15878
[3] Liu W, Liu S Y, Zhou T T and Wang L J 2009 *Opt. Express* **17** 19278
[4] Liu W, Liu S Y, Shi T L and Tang Z R 2010 *Opt. Express* **18** 20096
[5] Saleh B E A 1979 *Opt. Acta* **26** 777
[6] Kintner E C 1978 *Appl. Opt.* **17** 2747
[7] Yamazoe K 2010 *J. Opt. Soc. Am.* A **27** 1311
[8] Liu S Y, Liu W and Zhou T T 2011 *J. Micro/Nanolith. MEMS MOEMS* **10** 023007
[9] Flagello D G, Klerk J, Davies G, Rogoff R, Geh B, Arnz M, Wegmann U and Kraemer M 1997 *Proc. SPIE* **3051** 672
[10] Lohmann A W and Wirnitzer B 1984 *Proc. IEEE* **72** 889
[11] Hopkins H H 1953 *Proc. R. Soc. London* A **217** 408
[12] Zernike F 1934 *Physica* **1** 689
[13] Yu P, Qiu W and Pan D Z 2008 *IEEE Trans. Semicond. Manuf.* **21** 638
[14] Pati Y C and Kailath T 1994 *J. Opt. Soc. Am.* A **11** 2438