Depolarization effect of bandwidth in Mueller matrix imaging polarimetry

Weichao Du [a], Chuanwei Zhang [a], Xiuguo Chen [a], Weiqi Li [a], Shiyuan Liu [a,b,*]

[a] *State Key Lab of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan 430074, China*
[b] *Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China;*
** Contact author: shyliu@mail.hust.edu.cn*

Abstract: Obvious depolarization effect of bandwidth is observed in Mueller matrix imaging polarimetry (MMIP) with a dual-rotating compensator configuration. A method is proposed to correct the effect and to improve the measurement accuracy.
OCIS codes: (110.5405) Polarimetric imaging; (120.2130) Ellipsometry and polarimetry

1. Introduction

In recent years, Mueller matrix imaging polarimetry (MMIP) is introduced to measure the spatially dependent polarization properties of optical samples and optical systems, and the most common design of MMIP is to modify the traditional Mueller matrix polarimetty(MMP) with an imaging lens and a CCD camera in the analyzer arm [1, 2]. A xenon lamp with a monochromator is used as the light source to meet the demands of multiple wavelengths measurement. Compared with the traditional spectroscopic MMP, the MMIP can achieve the spatial distribution of polarization parameters of the sample in a single measurement. It has been applied in medicine, material and semiconductor industry. However, when the monochromator is used in the MMIP, the light diffracted by the grating monochromator has a finite bandwidth and thus different wavelengths are measured simultaneously by the detector. Generally, there is a trade-off between the bandwidth and the measurement intensity. The bandwidth is set to maintain a sufficient intensity received by the CCD camera at a high level, since the poor signal to noise ratio of the CCD camera will induce measurement errors. In this case, depolarization occurs due to the wavelength dependence of the optical properties of the sample [1]. More significantly, the depolarization effect will have an influence on the measurement accuracy, and it must be considered in the optical model. Several researchers have investigated the depolarization effect of polarimetric components such as polarizers and compensators in previous work [3, 4]. In this work, we focus on the depolarization effect of bandwidth in MMIP and demonstrate that this effect can induce significant measurement errors. Consequently, we propose a method to correct this effect and to improve the measurement accuracy.

2. Method

Figure 1 shows the diagram of the MMIP considered in our work, which is based on a dual-rotating compensator configuration [5]. Each pixel of the CCD camera can be regarded as an independent "MMP" and the well-established dual-rotating compensator method is used to calculate the Mueller matrix elements. However, the significant depolarization is found in this setup, owning to the finite bandwidth of the monochromator. The depolarization effect of a depolarizing Mueller matrix can be described by the depolarization index DI that is defined by [6]

$$DI = \left[\frac{\mathrm{Tr}(\mathbf{M}\mathbf{M}^{\mathrm{T}}) - m_{11}^2}{3m_{11}^2} \right]^{1/2}, \; 0 \le DI \le 1, \tag{1}$$

where m_{11} is the (1,1)th elements of the Mueller matrix $\mathbf{M}$, $\mathbf{M}^{\mathrm{T}}$ is the transposed matrix of $\mathbf{M}$, and Tr( ) represents the trace of matrix. DI=0 and DI=1 correspond to a totally depolarizing and non-depolarizing Mueller matrix, respectively.

In particular, a depolarizing system is optically equivalent to a system composed of a parallel combination of several non-depolarizing optical systems, since the optical modeling principle is based on the optical equivalence of the polarization sates. It can be further deduced that a depolarizing Mueller matrix can be expressed as the sum of various non-depolarizing Mueller matrix. Thus, when the depolarization induced by bandwidth is considered in the optical model, the depolarizing Mueller matrix induced by the finite bandwidth can be expressed as [7]

$$\mathbf{M} = \int \omega(\lambda)\mathbf{M}(\lambda)\,\mathrm{d}\,\lambda \tag{2}$$

where $\omega(\lambda)$ is a bandwidth function and $\int \omega(\lambda)\,\mathrm{d}\,\lambda = 1$. $\mathbf{M}(\lambda)$ is a non-depolarizing Mueller matrix with respective to the single wavelength λ.

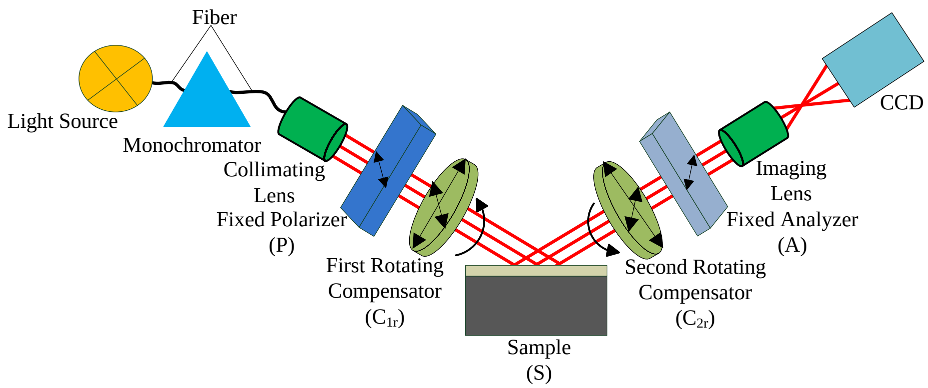

Fig. 1. Diagram of the dual-rotating compensator Mueller matrix imaging polarimeter.

3. Results

We apply the MMIP in optical critical dimension (OCD) metrology. The investigated sample is defined as an ideal one-dimensional Si grating which is characterized by a rectangle model with critical dimension (CD), grating height (Hgt), side-wall angle (SWA), and period (p). Nominal dimensions of the ideal grating sample are CD = 250nm, Hgt = 400nm, SWA = 90°, and p = 500nm. The wavelength is varied from 250nm ~ 800nm by the monochromator. The incident angle and azimuth angle are set as 60° and 0°, respectively. The theoretical Mueller matrix of the ideal grating sample is calculated by using rigorous coupled-wave analysis (RCWA). Figure 2 presents the *DI* of the sample varying with wavelengths under different bandwidths. We observe that the *DI* has an obvious decreasing with the bandwidth increment and is sensitive to the wavelength. When the value of *DI* is small, the strong depolarization occurs and large measurement errors will be induced by the depolarization effect.

The structural parameters of the sample are extracted by using a Levenberg-Marquardt algorithm which is based on nonlinear least-squares fitting of the Mueller matrix spectra with and without considering the depolarization effect of bandwidth. As can be observed from Figure 3, the extracted structural parameters of the sample have a significant deviation from the nominal values when the depolarization effect has not been considered, and the deviation increases with the increment of bandwidth. After incorporating the depolarization effect in the optical model, the extracted structural parameters are almost the same with nominal values which verifies the proposed correction method. It can be also observed that the value of CD is much more sensitive to depolarization effect of bandwidth than other parameters. For example, when the bandwidth is equal to 9nm, the CD has a 15.4nm deviation of the nominal value. In practice, the finite bandwidth of the monochromator is general larger than 3nm in order to maintain a sufficient intensity on the CCD camera in MMIP. In this case, the depolarization effect of bandwidth will induce large measurement errors, and the correction method must be performed to correct effect and to improve the measurement accuracy.

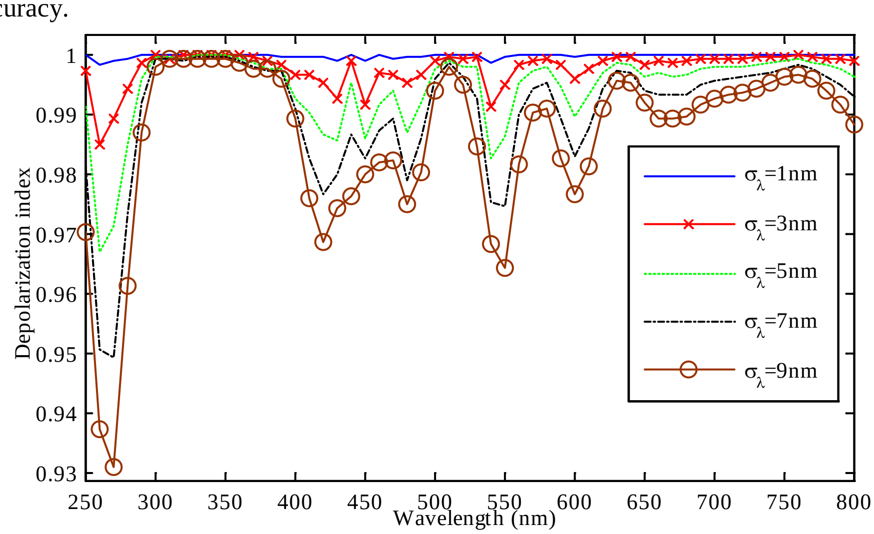

Fig. 2. Depolarization index of the sample varies with wavelengths under different bandwidths. σ_λ is the finite bandwidth of the monochromator.

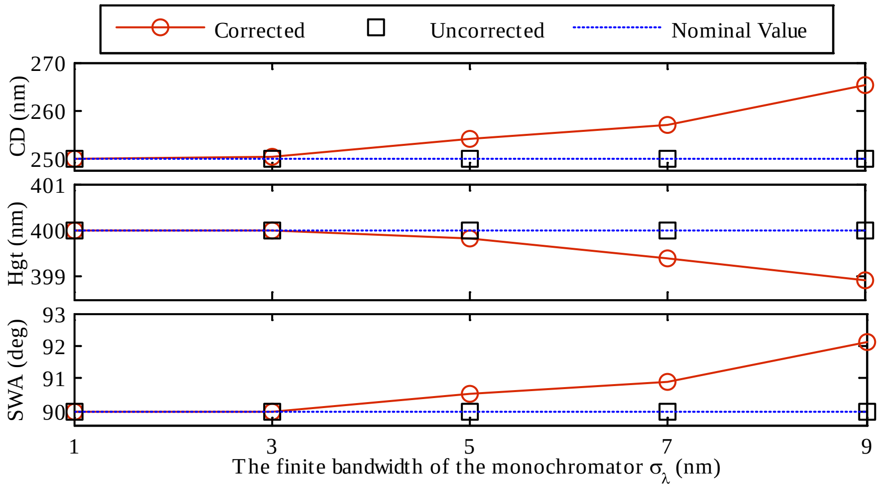

Fig. 3. The extracted structural parameters of the sample with and without considering the depolarization effect of bandwidth.

4. References

[1]. H. Fujiwara, *Spectroscopic Ellipsometry: Principles and Applications* (Wiley, 2007).

[2]. L. Jin, T. Hamada and Y. Otani et al, "Measurement of characteristics of magnetic fluid by the Mueller matrix imaging polarimeter," Opt. Eng. **43,** 181-185 (2004).

[3]. S.-M.F. Nee, "Effect of depolarization of polarimetric components on null ellipsometry," Thin Solid Films **313-314,** 90-96 (1998).

[4]. B. H. Ibrahim, S. B. Hatit and A. D. Martino, "Angle resolved Mueller polarimetry with a high numerical aperture and characterization of transparent biaxial samples," Appl. Opt. **48,** 5025-5034 (2009).

[5]. J. A. Zapien, R.W. Collins and R. Messier, "Multichannnel ellipsometer for real time spectroscopy of thin film deposition from 1.5 to 6.5 eV," Rev. Sci. Instrum. **71,** 3451-3460 (2000).

[6]. J. J. Gil and E. Bernabeu, "Depolarization and Polarization Indices of an Optical System," Opt. Acta **33,** 185-189 (1986).

[7]. X. G. Chen, C. W. Zhang and S. Y. Liu, "Depolarization effect from nanoimprinted grating structures as measured by Mueller matrix polarimetry," Appl. Phys. Lett. **103,** 151605 (2013).