Numerical analysis of transmission efficiency for parabolic optical fiber nano-probe

Wei Zhu,1 Tielin Shi,1,2 Zirong Tang,1,2,* Bo Gong,2 Guanglan Liao,2 and Shiyuan Liu2
1 Wuhan National Laboratory for Optoelectronics, Huazhong University of Science and Technology, Wuhan 430074, China
2 State Key Lab of Digital Manufacturing equipment and technology, Huazhong University of Science and Technology, Wuhan 430074, China
*zirong@mail.hust.edu.cn

Abstract: Theoretical calculations are performed for the transmission efficiencies of parabolic nano-probes with different shapes, based on the finite element method. It shows that the transmittance will fluctuate dramatically with the variation of either wavelength or probe shape, and the efficiency could be rather high even at long wavelengths. Subsequently, we thoroughly investigate this phenomenon and find that these fluctuations are due to the joint effect of light propagating modes and surface plasmon polaritons modes. It indicates that high transmittance can be achieved with the selection of appropriate wavelength and probe structure.

©2013 Optical Society of America

OCIS codes: (000.4430) Numerical approximation and analysis; (060.2310) Fiber optics; (120.4640) Optical instruments; (180.5810) Scanning microscopy.

References and links

1. Introduction

Scanning near-field optical microscopy (SNOM) is a crucial candidate to reach the submicron resolution, due to its capability of breaking through the diffraction limit. For SNOM, the optical fiber nano-probe is a key component, whose properties mainly depend on the probe shape parameters, such as the cone angle and tip diameter. In order to characterize the probe properties, sophisticated and costly instruments are often demanded for experimental investigations [1–3]. On the other hand, the numerical analysis on the probe performance is an alternative approach due to the low cost and accessibility. In the reported literatures, enormous theoretical works have been fulfilled [4–8], while these efforts were mainly concentrated on the probes with linear shapes and systematic simulations considering various probe shapes have not been implemented yet. Some previous works have shown that parabolic shape fiber probes can give higher transmittance comparing to linear ones [9,10]. Though the model has been set up for parabolic probe [11], the core and cladding were treated as a whole, which is only suitable for the condition of the weak waveguide mode with negligible refractive index difference. When this difference can’t be neglected, the model has to be improved, and the core and cladding have to be treated separately. In this work, the parabolic fiber probes of different nonlinear shapes, which can be fabricated by the dynamic selective etching method [12], are modeled with the core of nonlinear shape protruded from the cladding. The transmission efficiencies of the probes are also investigated based on the numerical simulations.

2. Theoretical modeling

The geometrical feature of the parabolic probe is schematically illustrated in Fig. 1 with the axial symmetry, where the probe is composed of the linear and nonlinear parts. The linear shape is the curvature with a cone angle of 120° nearby the apex, and the nonlinear shape is the section of parabola tangential to the linear curvature. The cone angle at the input port is defined as $\theta$. The input port fiber diameter $D_c$ is 7 $\mu$m, and the aperture diameter $D_a$ is 100 nm. The model is constructed in the Cartesian coordinate. Due to the nonlinearity of probe shape, the metallic coating thickness is derived from the transversal shift of probe curvature, which is defined as T of 200 nm. The metal adopted here is the aluminum with dielectric constant taken from Smith et al. [13].
As reported previously, the highest transmission efficiency will be obtained through the injection of linearly polarized fundamental (HE$_{11}$) mode [4,14]. Since the fundamental mode field of step-index fiber is the Bessel distribution with the energy mainly concentrating in the central region. Therefore, we simply define the light source as a Gaussian beam linearly polarizing along x direction with the waist radius of w. Thus the light distribution on the input boundary can be expressed by Eq. (1),

$$ E = A_0 \exp\left(-\frac{x^2 + y^2}{w^2}\right) $$  \hspace{1cm} (1)

where the amplitude $A_0$ is normalized to 1, the value of $w$ is equal to the Marcuse Radius determined by Eq. (2) [15],

$$ \frac{w}{a} = 0.65 + \frac{1.619}{V^{1/3}} + \frac{2.879}{V^8} $$  \hspace{1cm} (2)

where a is the fiber radius of 3.5 μm, and V is the normalized cutoff frequency determined by Eq. (3),

$$ V = \frac{2\pi a}{\lambda} NA $$  \hspace{1cm} (3)

where NA is the fiber numerical aperture of 0.25.

As inferred from Eq. (3), when V is larger than 2.405 with the decrease of the wavelength, the higher order modes are able to be introduced [16]. In this study, we consider only HE$_{11}$ mode since it contributes most to the effective improvement of the power transmission and the effects of higher order modes are neglected. The corresponding Maxwell equations of the model are numerically solved by the finite element method (FEM), and all the calculation is performed on the commercial FEM software COMSOL Multiphysics.

3. Results and discussion

In SNOM regime, the transmission efficiency is defined as the ratio of the output power intensity ($I_{out}$) to input power intensity ($I_{in}$), namely, $I_{out} / I_{in}$ and the power intensity is equivalent to the Poynting vector. We calculate the transmittances of parabolic probes with different angle of $\theta$ at various wavelengths, and the curves are plotted in Fig. 2.
As shown in Fig. 2, the transmission efficiency depends on both the probe shape and wavelength, and fluctuates with the change of wavelength for a certain probe shape. With the variation of probe shape and wavelength, the predicted transmittance ranges from the lowest magnitude of $10^{-8}$ to the highest magnitude of $10^{-1}$. Although systematic investigations on the transmittance versus wavelength have not been experimentally conducted for parabolic probes in reported literatures, there are some experimental studies concerning the transmission efficiency at a fixed wavelength [9, 10]. At the wavelength of 633 nm [9], the transmission efficiency was reported to range from $3.8 \times 10^{-5}$ to $6.5 \times 10^{-4}$ with the tip diameter of 80 nm, while it was in the range of $1 \times 10^{-7}$ to $3 \times 10^{-5}$ with the same tip diameter at the wavelength of 514 nm [10]. This indicates that optimal transmission efficiency can be achieved by the proper selection of both probe shape and wavelength, which is in consistent with our theoretical prediction.

Moreover, it shows that all the efficiencies decrease gradually with the increase of wavelength on the whole, namely that for every probe, the efficiency fluctuates up and down with a gradual diminishing trend. The highest transmission efficiency of all the probes appears when $\theta$ is 70° at the wavelength of 500 nm and the lowest efficiency appears when $\theta$ equals 40° at the wavelength of 1350 nm. In order to understand this phenomenon we subsequently investigate the light propagation of these two situations.

Figure 3 shows the electric field and power intensity distributions of the central x-z and y-z cross sections.
In the situation with the highest efficiency, as indicated in Figs. 3(a) and 3(b), the light waves propagate along the probe with a part of waves reflected back and the others transmitting through the probe aperture. While for the lowest efficiency, as illustrated in Figs. 3(c) and 3(d), the waves are mostly reflected back and then concentrate to a location farther from the aperture, leading to the poor throughput of light intensity. Hence, the huge distinction between situation of the highest and lowest efficiencies is related to the propagating characteristics of lights inside the dielectric region.

It is known that the SNOM probe’s transmission regime is the light wave propagation through the hollow metal waveguide filled with dielectric medium [17,18], there will be different modes in the dielectric cone with the change of diameter. With the decreasing of diameter, the higher-order modes will vanish and only the fundamental mode propagates in the probe. Subsequently, when the waves reach the region of the cutoff diameter, even the fundamental mode will be transformed to the non-propagating mode that attenuates faster than exponentially [19]. On the other hand, there will be surface plasmon polaritons (SPPs) with the intrinsic energy dissipation in the metal layer [20]. Therefore the power dissipation is attributed to the losses of modes in propagation regime, the dramatic attenuation of cutoff regime and the transformation of SPP regime. The cutoff diameter can be estimated through $D_c = \frac{\lambda}{2n_1}$ where $n_1$ is the core refractive index and $\lambda$ is the wavelength [21]. The cutoff...
region of different probes are the same except when \( \theta \) equals 20° with a negligible nonlinear curvature. Hence, the influence of cutoff region can be excluded. We then calculate the transmission ratio of power intensity of the cross section at the cutoff diameter to the input port to obtain the transmission properties of the propagating regime. Although the SPP modes may be excited on the metallic and dielectric interface, the fields distribute mostly on the central area, leading to a limited effect of SPP. In addition, the SPP has its intrinsic attenuation along the interface, therefore its influence on the transmission will not be considered. Figure 4 plots the transmission ratio for the propagating and cutoff regions at the wavelength of 630 and 1550 nm, which are commonly adopted in communication industry.

![Fig. 4. Transmission ratios of power intensity for probes with various angles: (a) ratio of the cutoff diameter’s cross section to the input port; (b) ratio of the output port to the cutoff diameter’s cross section.](image)

Figure 4(a) plots the ratio of the cross section at the cutoff diameter to the input port, where the highest and lowest efficiencies are about 0.5 and 0.003 respectively, and the ratios fluctuate with the angle of \( \theta \) at the wavelength of both 630 and 1550 nm. It is obvious that the probe shape significantly affects the transmission ratio as has been verified by the sharply fluctuations with different angle of \( \theta \). In fact, in the propagating region, due to the metal and dielectric interface, a part of light waves travel back after one or multiple reflections, while the others will propagate along the cutoff region. During this process, this region also plays a role similar to a resonant cavity, which is determined by the probe shape and wavelength, leading to the fluctuations of the transmittance. Then we calculate the ratio of power intensity of the output port to the cross section at the cutoff diameter, as plotted in Fig. 4(b), where the ratios also fluctuate at the wavelength of 630 and 1550 nm respectively.

In this region, since the propagating mode has reached cutoff regime, the SPP modes will be responsible for the fluctuations as it is non-cutoff even in sub-wavelength waveguides and can be converted into the modes propagating in dielectric medium [22,23]. In SPP regime, the excited surface plasmon waves (SPWs) will extend from the metal layer to the dielectric medium, and the penetrating depth is defined by Eq. (4),

\[
\delta_d = \frac{1}{k_0} \sqrt{\frac{\varepsilon_d + \varepsilon_m^\alpha}{\varepsilon_d^2}}
\]

where \( \varepsilon_d = 2.1 \) is the permittivity of the core, \( \varepsilon_m^\alpha \) is the real part of the permittivity of aluminum which can be inferred from the previous publication [13]. The calculated depth is 350 and 1870 nm at the wavelength of 630 nm and 1550 nm respectively, and it is obvious that parts of the SPWs will extend to the glass core due to the large penetrating depths relative to the cutoff region. In this region, the light waves are highly concentrated, thus a lot of light waves can be transformed to SPWs, and then parts of the SPWs will stretch into the dielectric medium, leading to the increase of transmission efficiency.
For the highest ($\lambda = 630$ nm, $\theta = 70^\circ$) and lowest ($\lambda = 1550$ nm, $\theta = 60^\circ$) ratios in Fig. 4(b), we calculate the electric field and power intensity distributions of each situation, and the field distributions are magnified for the region marked with the red frames. Figures 5(a) and 5(b) correspond to the highest ratio and Figs. 5(c) and 5(d) correspond to the lowest ratio.

![Electric field and power intensity distributions](image)

Fig. 5. (a) Electric filed and (b) power intensity distributions of the central cross section for $\lambda = 630$, $\theta = 70^\circ$; (c) electric field and (d) power intensity distributions of the central cross section for $\lambda = 1550$, $\theta = 60^\circ$. The 2D plots are the magnified field distributions of the area in the red frames.

As illustrated in Figs. 5(a) and 5(b), the fields in the cutoff region exhibit higher magnitudes compared to the adjacent area of propagating region and are highly concentrated at the aperture. It shows that the fields near the aperture edges are of higher intensities than those in the middle of aperture, leading to a high level of transmission ratio. We attribute the
field distribution of high magnitudes in the cutoff region to the penetration of SPP into the
dielectric medium, which can be also verified since the electric fields of the cutoff region in
x-z central plane are obviously higher than those in y-z central plane. Since the input source is
polarized along x axis, the light in x-z central plane propagates like the p-polarized wave
which is capable of exciting the SPP mode. While in y-z central plane, the light behaves
similar to the s-polarized wave which is ineffective to the SPP excitation [20,24]. On the
other hand, as shown in Figs. 5(c) and 5(d), the field magnitudes in the cutoff region are much
weaker than the adjacent part of propagating region, and the fields at the aperture are rather
weak, which leads to the poor transmission ratio. Under this situation, SPP effect is not
apparent due to the ineffective excitation of SPP mode. In terms of SPP, the excitation
condition is determined by Eq. (5),

\[ k_{spp} = \frac{2\pi}{\lambda} n_i \sin \alpha \]  

where \( k_{spp} \) is the wave vector of SPP, \( \alpha \) is the incident angle of light wave from the dielectric
medium to the metal surface. When the probe shape is changed, \( \alpha \) will be changed
accordingly, hence, the excitation of SPP is determined by the probe structure and
wavelength. Consequently, the fluctuations of the transmission ratio in cutoff region are
attributed to the selective excitation of SPP modes.

In general, the propagating regime and SPP modes in cutoff region will both cause the
fluctuations of transmission ratio, and are both affected by the probe shape and wavelength.
Therefore, the fluctuations of the transmittance are attributed to the joint effect of propagating
and SPP modes.

4. Conclusions

We have systematically investigated the propagation properties of parabolic nano-probes, and
find that the efficiencies are fluctuated and significantly affected by the probe shape and the
wavelength. This is explained by the compound effect of the propagating modes in the
propagation region and the SPP modes in cutoff region. In the propagating regime, the
fluctuations are derived from the resonant cavity effect of light waves, while in the SPP
regime, they are attributed to the selective excitation of SPP mode. This work provides the
design guidelines for optical fiber nano-probes in SNOM application, and the highest
transmission efficiency can be obtained through optimal design of probe shape and
appropriate selection of wavelength.

Acknowledgment

This work is financially supported by National Science Foundation of China (No. 51275195)
and the National Instrument Development Specific Project of China (No. 2011YQ160002).