



## Full Length Article

# Angle-resolved ellipsometric analysis of polishing-induced subsurface damages in calcium fluoride crystals for ultra-precision manufacturing

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## ABSTRACT

Calcium fluoride (CaF<sub>2</sub>), renowned for its exceptional optical and mechanical properties, is a critical material in advanced semiconductor and optical manufacturing. However, tribological processes such as polishing introduce subsurface damages (SSDs), including micro-cracks and lattice dislocation, which degrade functional performance and reduce service life. This study presents a novel, nondestructive method for SSD inspection using single-shot angle-resolved ellipsometric spectra (ARES) acquisition and analysis. The effects of SSD layer thickness and surface roughness on ARES are systematically evaluated through the effective medium approximation. Finite-difference time-domain simulations demonstrate the high sensitivity of Ψ-ARES (p-/s- amplitude ratio) to nanoscale SSD variations. An ARES measurement system employing back-focal-plane imaging was developed to validate the approach on CaF<sub>2</sub> samples subjected to varying polishing conditions. The results showed good agreement with that of atomic force microscopy, white light interferometry and transmission electron microscopy. This work establishes angle-resolved ellipsometry as a powerful tool for nondestructive subsurface damage evaluation, enabling improved polishing processes and enhanced performance of ultra-precision manufacturing.

## 1. Introduction

Calcium fluoride (CaF<sub>2</sub>), renowned for its exceptional optical and physical properties, is widely used as a critical window material in deep ultraviolet lithography and the aerospace industry. These properties include high optical isotropy, superior transmission, temperature resistance, and corrosion resistance [1–5]. High surface quality, encompassing well controlled surface roughness (SR) and subsurface damage (SSD) [6–8], is essential in these industries. Among these, SSD is a crucial indicator of surface quality, as it significantly impacts the functional performance and service life of lithography systems and semiconductor devices [9–11]. SSD forms during mechanical processing due to stress and chemical reactions, manifesting in internal fractures, amorphous structures, dislocations, layering, lattice distortions, and residual stress [12–14]. Accurate and efficient characterization of SSD is critical for achieving the required surface quality in optical crystals.

### 1.1. Current SSD characterization methods

Current SSD characterization methods are often destructive, even

when partially contactless optical tools are employed. For example, white light interferometry (WLI) can nondestructively measure SR but becomes destructive for SSD inspection, requiring critical sample pre-processing, such as surface etching. Chu et al. [15] used WLI to analyze SSD morphology in molten silicon by etching surfaces, establishing a relationship between SSD and resonator quality factor. Similarly, Guo et al. [16] employed transmission electron microscopy (TEM) to investigate the tough-brittle transformation mechanism in CaF<sub>2</sub> micro-cutting processes. Currently, TEM is a typical method for characterizing SSD [17–19]. However, destructive methods reduce reliability and efficiency because sample surface layers must be destructed prior to measurement.

Optical coherence tomography (OCT) and scanning acoustic microscopy (SAM) are typical nondestructive techniques for SSD characterization [20,21]. However, OCT struggles to achieve the nanoscale resolution required for precise SSD measurements [20]. SAM is constrained by environmental requirements due to its reliance on acoustic coupling via a water medium [22]. Emerging optical methods, including Raman spectroscopy, X-ray diffraction, and ellipsometry, have been proposed for nondestructive surface quality evaluation. Raman spectroscopy, for instance, provides information on lattice vibrations influenced by microstructure changes and residual stress [23–25]. Zhou et al.

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Nomenclature			
$\Delta$	Phase retardance	$d_{\text{EMA}}$	EMA equivalent thickness
$\varepsilon$	Dielectric constant	EMA	Effective medium approximation
$\lambda$	Wavelength	FDTD	Finite-difference time-domain
$\Psi$	Amplitude ratio	$G_q$	Gradient angle of $\theta_q$ in $\Delta$ -ARES
$\sigma$	Root-mean-square surface roughness	$h$	Average layer thickness in EMA
$\theta$	Incident angle	$n$	Refractive index
$\theta_B$	Brewster angle	NA	Numerical aperture
$\theta_q$	Quasi-Brewster angle	OCT	Optical coherence tomography
AFM	Atomic force microscope	q-BAT	Quasi-Brewster angle technique
ARE	Angle-resolved ellipsometry	$r$	BFP radius
ARES	Angle-resolved ellipsometric spectra	$S_B$	Angular shift of $\theta_B$ in $\Psi$ -ARES
BFP	Back-focal plane	$S_q$	Angular shift of $\theta_q$ in $\Delta$ -ARES
CaF <sub>2</sub>	Calcium fluoride	SR	Surface roughness
$d_1$	Surface roughness layer thickness	SSD	Subsurface damage
$d_2$	Subsurface damage layer thickness	TEM	Transmission electron microscopy
		VASE	Variable angle spectroscopic ellipsometry
		WLI	White light interferometry

[26] demonstrated the plastic deformation mechanism of gadolinium gallium garnet during atomic-scale grinding using Raman spectroscopy. However, conventional Raman signals are weak for nanoscale SR layers, and tip-based Raman spectroscopy, while effective for film thicknesses of 30 nm, incurs high costs [27]. X-ray diffraction has been used to evaluate SSD in CaF<sub>2</sub> grinding processes, correlating micro-cracks with diffraction curve patterns [28]. Yet, complex equipment and sample limitations (e.g., powders) hinder its broad application [25]. These limitations underscore the urgent need to develop new nondestructive and high-precision SSD characterization methods.

### 1.2. Ellipsometry combined with advanced modeling

With its sub-nanometer sensitivity, Ellipsometry has recently emerged as a promising tool for SSD inspection. Ellipsometry can infer SSD depth through advanced optical modeling, e.g., effective medium approximation (EMA) by measuring surface layer thickness and refractive index. Wang et al. [29] introduced the quasi-Brewster angle technique (q-BAT) to measure SR and SSD depths in optical crystals simultaneously. Yao et al. [30,31] applied q-BAT with variable angle spectroscopic ellipsometry to evaluate the surface quality of polished Lu<sub>2</sub>O<sub>3</sub> and yttrium aluminum garnet crystals. These studies emphasized the significance of angular spectra of the phase retardance ( $\Delta$ ) between *p*- and *s*- polarized light by simply correlating the angular shift and spectral curve slope at  $\Delta = 90^\circ$  to SSD and SR separately. However, q-BAT often leads to confusion, as it ignores the angular shifts caused by SR, resulting in reduced accuracy. In addition, the theoretical foundation of q-BAT has yet to be understood appropriately.

Finite-difference time-domain (FDTD) simulations have validated EMA's reliability. For example, Yu et al. [32] analyzed SiO<sub>2</sub> thin films using FDTD, and established a functional relationship of EMA equivalent thickness ( $d_{\text{EMA}}$ ) in terms of root-mean-square SR ( $\sigma$ ) and average layer height ( $h$ ):  $d_{\text{EMA}} = \sigma + 0.8h$ . Huang et al. [33] demonstrated that EMA accuracy for SR evaluation improves as the ratio of  $\sigma$  to light wavelength ( $\lambda$ ) decreases. Fung et al. [34] explored EMA's applicability for randomly rough surfaces regarding  $\sigma/\lambda$  under normal incidence light and highlighted the need for studies under oblique incidence conditions. While these studies focus on SR evaluation, research on SSD characterization using EMA remains limited.

### 1.3. Challenges and contribution

The following key challenges remain unresolved: 1) The theoretical validity of q-BAT for SSD evaluation needs further examination, particularly in separating SR and SSD contributions. The inability to

distinguish and quantify SSD and SR layer thicknesses using  $\Delta$  angle-resolved ellipsometric spectra ( $\Delta$ -ARES) compromises accuracy. 2) Existing ellipsometry systems rely on variable-angle scanning for ARES acquisition, significantly reducing measurement efficiency [35].

In this work, a novel angle-resolved ellipsometry (ARE) method is developed for SSD evaluation. First, limitations of q-BAT are addressed using EMA, and a new  $\Psi$ -ARES-based indicator,  $S_B$ , is proposed for SSD thickness quantification. Second, the enhanced q-BAT method is validated through FDTD simulations under oblique incidence, and a quantitative mapping function between SSD and  $S_B$  is established. Third, an ARE system is developed and applied to evaluate SSA in CaF<sub>2</sub> crystals, under varying polishing conditions. Validation experiments using WLI and TEM confirm the accuracy of the proposed method. This work highlights the potential of ARE as a powerful tool for nondestructive SSD evaluation, advancing surface quality inspection techniques.

## 2. Method

### 2.1. Principle of q-BAT

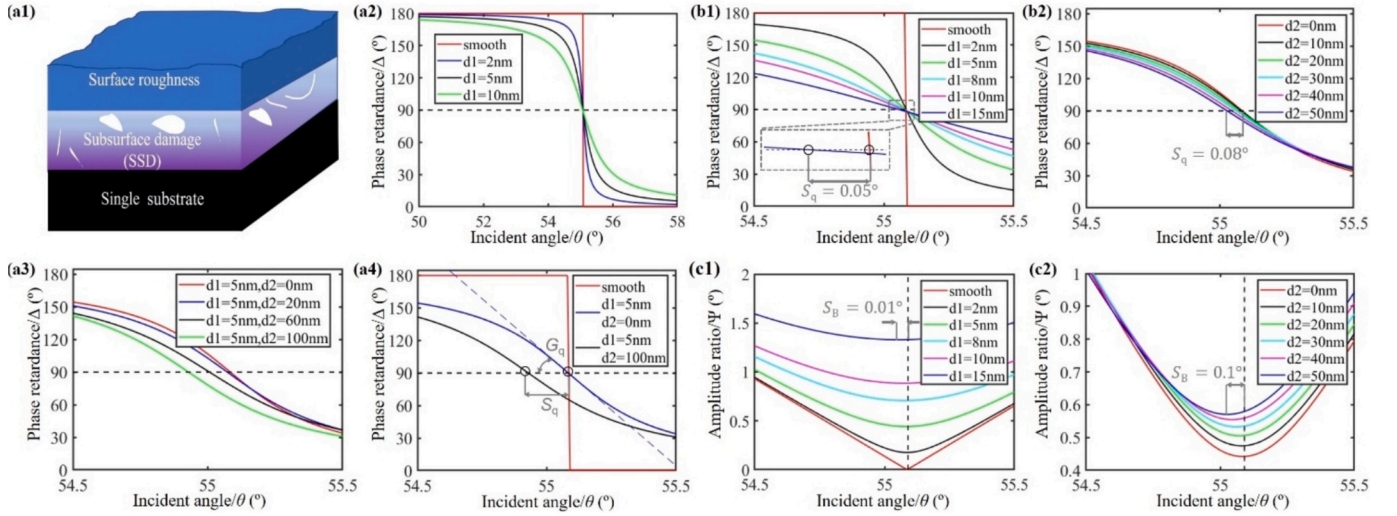
Ellipsometry characterizes a sample by measuring the changes in the polarization state of incident light upon reflection. The polarization state is typically expressed as the ratio of the wave component parallel (*p*-polarized) and perpendicular (*s*-polarized) to the plane of incidence. This ratio is represented using two parameters: the amplitude ratio  $\Psi$  and their phase retardance  $\Delta$ , defined as:

$$\tan\Psi \cdot \exp(i\Delta) = r_p/r_s \quad (1)$$

where  $r_p$  and  $r_s$  are the Fresnel reflection coefficients for *p*- and *s*-polarized light, respectively. The Brewster angle ( $\theta_B$ ) is the incidence angle at which the reflected amplitude of *p*-polarized light vanishes [36], causing  $\Psi$  to reach its minimum value (zero for ideal surfaces). In contrast, the quasi-Brewster angle ( $\theta_q$ ) is the incidence angle where  $\Delta = 90^\circ$  [29]. As illustrated in Fig. 1(a), q-BAT enables surface quality evaluation based on the following principles:

- 1) SSD layer thickness determines the angular shift,  $S_q$ , of the  $\Delta$ -ARES curve at  $\theta_q$ ;
- 2) SR layer thickness influences the local gradient angle,  $G_q$ , of the  $\Delta$ -ARES curve at  $\theta_q$ .

The theoretical foundation of q-BAT lies in the EMA theory [37], which models SR or SSD as homogeneous materials with effective optical refractive indices. EMA provides a link between layer



**Fig. 1.** EMA analysis of ARES regarding the variation of SR and SSD. (a1) Illustration of SR and SSD; (a2-a3) effect of SR and SSD on  $\Delta$ -ARES, respectively; (a4)  $G_q$  and  $S_q$  on  $\Delta$ -ARES derived from a combined effect of SR and SSD; (b1-b2) sensitivity of  $S_q$  to SR and SSD derived from  $\Delta$ -ARES; (c1-c2) sensitivity of  $S_B$  to SR and SSD derived from  $\Psi$ -ARES.  $d_1$  and  $d_2$  represent SR and SSD layer thicknesses, respectively. In (b2) and (c2),  $d_1 = 5\text{nm}$ .  $G_q$ ,  $S_q$ , and  $S_B$  correspond to the gradient angle, angular shift of  $\theta_q$  in  $\Delta$ -ARES, and the angular shift of  $\theta_B$  in  $\Psi$ -ARES, respectively.

characteristics, e.g., thickness and void percentage, and optical properties, e.g., refractive index ( $n$ ), as described by:

$$f_a \frac{\varepsilon_a - \varepsilon}{\varepsilon_a - 2\varepsilon} + 1(-f_a) \frac{\varepsilon_b - \varepsilon}{\varepsilon_b - 2\varepsilon} = 0 \quad (2)$$

where  $\varepsilon = n^2$  is the effective dielectric constant,  $\varepsilon_a$  and  $\varepsilon_b$  are the dielectric constants of materials  $a$  and  $b$ , and  $f_a$  is the volume fraction of material  $a$ . For SSD layers,  $f_a$  represents the fraction of defects, such as cracks and pores and others.

An EMA-based q-BAT simulation was performed using MATLAB. The refractive index of air was set to 1, and the probing light wavelength was 632.8 nm. The refractive index of the  $\text{CaF}_2$  substrate was 1.433, modelled using the Sellmeier equation [38]. The SR and SSD layers were modelled with 50 % and 1 % volume fractions, respectively. Their thicknesses, denoted as  $d_1$  (SR) and  $d_2$  (SSD), were varied to observe their effects on  $\Delta$ -ARES. Fig. 1(a2) indicates that increasing SR thickness reduces  $G_q$  in the  $\Delta$ -ARES curve. Fig. 1(a3) indicates that with a fixed SR thickness of 5 nm (typical for ultra-precision polishing), increasing SSD thickness increases  $S_q$  magnitude. Therefore, as illustrated in Fig. 1(a4), q-BAT resolves SR and SSD thicknesses independently by analyzing  $G_q$  and  $S_q$ , respectively.

## 2.2. $\Psi$ -ARES-enhanced q-BAT

Despite its utility, the general q-BAT method has limitations. For example, the simulation results in Fig. 1(b1) show that as the SR layer thickness increased to 15 nm, a  $S_q$  of  $0.05^\circ$  was also observed. This value is equivalent to the  $S_q$  ( $0.08^\circ$ ) caused by an SSD of 50 nm thick, as presented in Fig. 1(b2). Both SR and SSD layer thicknesses in these simulations are commonly encountered in ultra-precision polishing practices [26,39,40]. The overlapping  $S_q$  values demonstrate a coupling effect, where  $S_q$  correlates with both SR and SSD, making SSD evaluation based solely on  $S_q$  unreliable. Additionally, cooperative evaluation using both  $S_q$  and  $G_q$  is time-consuming and prone to errors.

To address these issues, we propose a modified q-BAT method based on EMA analysis. The enhanced method replaces  $\Delta$ -ARES with  $\Psi$ -ARES for SSD evaluation, as shown in Fig. 1(c). Under the same surface conditions as above, i.e., SR varying from 0 to 15 nm and SSD varying from 0 to 50 nm,  $S_B$  (derived from  $\Psi$ -ARES) exhibits distinct behaviors:

- 1) Fig. 1(c1) indicates that  $S_B$  remains nearly constant ( $0.01^\circ$ ) as SR increases, indicating insensitivity to SR;
- 2) Fig. 1(c2) indicates that  $S_B$  increases significantly (up to  $0.1^\circ$ ) as SSD increases, demonstrating high sensitivity to SSD.

These results establish  $\Psi$ -ARES-resulted  $S_B$  as a more effective indicator for SSD evaluation than  $S_q$ . Based on this finding, therefore, this study combines  $S_B$  for SSD evaluation and  $G_q$  for SSD assessment as shown in Table 1, providing a more reliable approach for surface quality evaluation.

## 2.3. FDTD simulation

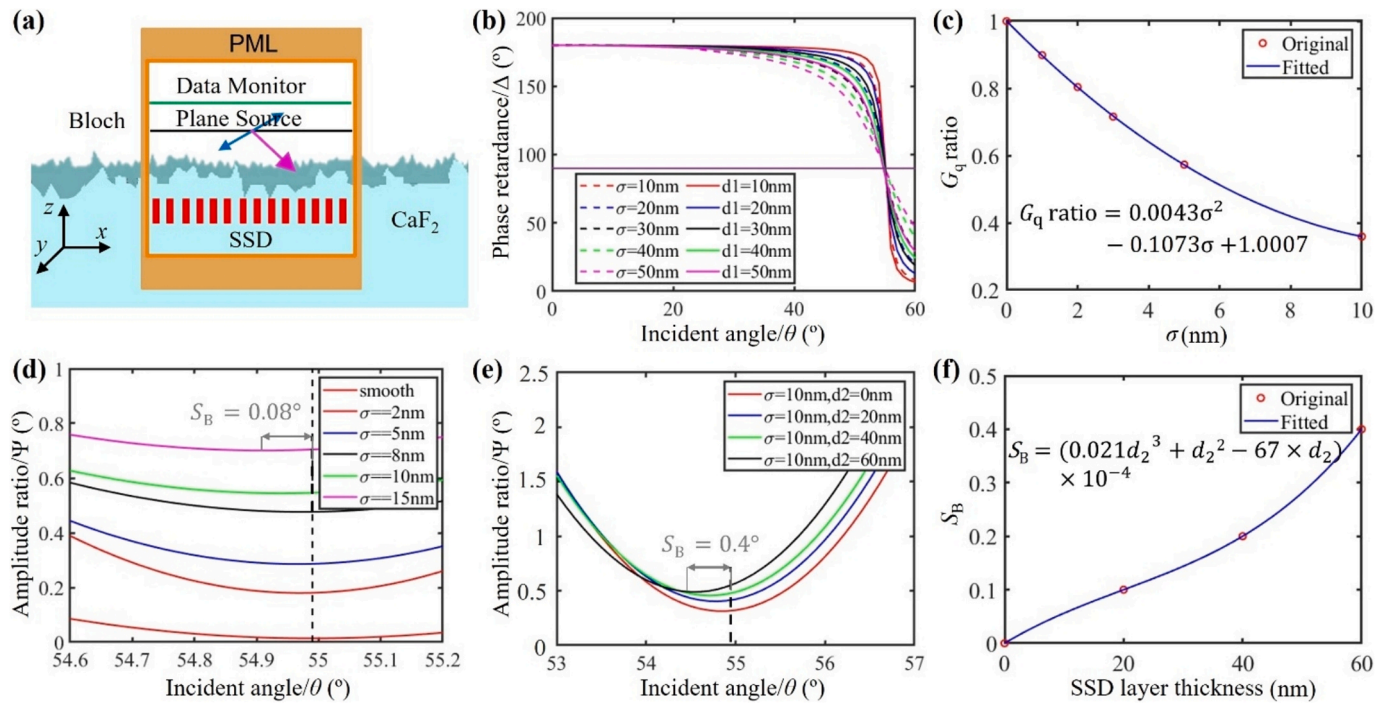
FDTD simulations were conducted to verify the feasibility of using  $\Psi$ -ARES for evaluating crystal SSD under oblique incidence, as well as the conclusions above. To enable simulations at multiple incidence angles, the plane wave Bloch/periodic boundary conditions were replaced by Broadband Fixed Angle Source Technique (BFAST). BFAST also replaced the Bloch boundary conditions in the  $x$  and  $y$  directions. The refractive index of  $\text{CaF}_2$  material was set to 1.433. The geometric model used in the FDTD simulation is illustrated in Fig. 2(a), with corresponding settings and parameters listed in Table 2.

Fig. 2(b) shows the FDTD results (dotted lines) compared with EMA results (solid lines). Both methods produced consistent results, with identical  $G_q$  and  $S_q$  values, validating the coupling phenomenon observed in Section 2.2. Specifically,  $S_q$  correlates equally with SSD and SR, making it insufficient to distinguish between them. Fig. 1(a) demonstrates that  $G_q$  correlates strongly with SR. To quantify this, Fig. 2(c)

**Table 1**  
Indicators of modified q-BAT for surface quality characterization.

Symbol	Definition	Object of evaluation
$\theta_q$	Quasi-Brewster angle, the incidence angle corresponding to $\Delta = 90^\circ$	
$\theta_B$	Brewster angle, the incidence angle at the minimum of $\Psi$ -ARES	
$S_q$	Angular shift of $\theta_q$ in $\Delta$ -ARES	SSD*
$G_q$	Gradient angle of $\theta_q$ in $\Delta$ -ARES	SR
$S_B$	Angular shift of $\theta_B$ in $\Psi$ -ARES	SSD

\*Due to the high sensitivity of  $S_q$  to SR,  $S_B$  found in the  $\Psi$ -ARES replaces  $S_q$  for SSD evaluation in our study.



**Fig. 2.** FDTD simulation results. (a) FDTD geometric model with SR and SSD (red, enlarged for clarity); (b) effect of SR layer root-mean-square roughness  $\sigma$  on  $\Delta$ -ARES obtained by FDTD (solid lines) compared to that of EMA (dashed lines); (c) functional mapping of  $G_q$  to SR- $\sigma$  ( $G_q$  ratio relative to that of the ideal surface is calculated for practice convenience); (d, e) effect of SR and SSD on  $\Psi$ -ARES, respectively (zoomed  $53^\circ$  to  $57^\circ$  for clarity) for sensitivity analysis. (f) functional mapping of  $S_B$  to SSD thickness  $d_2$  ( $S_B$  relative to that of the ideal surface with SR  $\sigma = 10\text{nm}$  is calculated for convenience).

**Table 2**  
Parameter settings of FDTD simulations.

	Parameter	Value
Boundary	x, y direction	BFAST
	z direction	PML
	Probing Light	Type
Probing Light	Wavelength	632.8 nm
	Polarization angle	0 or $90^\circ$
	Angle phi	$0^\circ$
	Amplitude	1
	Phase	0
Monitor	Type	Frequency domain field profile
SR Layer	Side length	1.5 $\mu\text{m}$
		2–50 nm
SSD Layer	Side length	10 nm
	Height	0–60 nm
	Number of rectangle void	10
	Pitch	10 nm
Substrate	Side length	3 $\mu\text{m}$

plots  $G_q$  as a function of root-mean-square SR ( $\sigma$ ), showing a quadratic relationship that allows for the quantitative evaluation of SR using  $G_q$ .

Fig. 2(b) shows the FDTD results (dotted lines) compared with EMA results (solid lines). Both methods produced consistent results, with identical  $G_q$  and  $S_q$  values, validating the coupling phenomenon observed in Section 2.2. Specifically,  $S_q$  correlates equally with SSD and SR, making it insufficient to distinguish between them. Fig. 1(a) demonstrates that  $G_q$  correlates strongly with SR. To quantify this, Fig. 2(c) plots  $G_q$  as a function of root-mean-square SR ( $\sigma$ ), showing a quadratic relationship that allows for the quantitative evaluation of SR using  $G_q$ .

The incidence angle range of  $53$  to  $57^\circ$  with a spacing of  $0.1^\circ$  was selected to optimize the simulation precision and efficiency. A quadratic polynomial was used for the local fitting of  $\Psi$ -ARES ( $54.6^\circ$ – $55.2^\circ$ ) to assess the impact of SSD layer thickness.

Regarding SR sensitivity, Fig. 2(d) shows the influence of SR layer thickness on  $\Psi$ -ARES. For  $\sigma$  ranging from 2 nm to 15 nm (consistent with

Fig. 1(c1)),  $S_B$  remains nearly constant ( $0.08^\circ$ ), confirming that  $S_B$  is insensitive to SR, as concluded in Fig. 1(c1).

Regarding SSD sensitivity, an array of rectangular void structures representing SSD (“etch” in FDTD with a refractive index of 1) was added below the geometric model’s rough surface, as presented in Fig. 2(a). The SSD layer thickness was varied from 0 to 60 nm (0 represents no SSD). Fig. 2(e) illustrates that  $S_B$  increases significantly with SSD layer thickness ( $S_B = 0.4^\circ$  for a 60 nm SSD layer), consistent with the results in Fig. 1(c2).

Fig. 2(f) plots the SSD layer thickness (0–60 nm) against  $S_B$ , where  $\sigma = 10\text{nm}$ . A quadratic relationship between SSD thickness and  $S_B$  is observed, enabling the quantitative evaluation of SSD. Similarly, Fig. 2(c) demonstrates a quadratic relationship between SR and  $G_q$ , confirming the potential of  $G_q$  and  $S_B$  for separate evaluations of SR and SSD.

The FDTD simulation results confirm the feasibility of  $\Psi$ -ARES for evaluating SSD under oblique incidence conditions. Key findings include:

- 1) SR and SSD layer thicknesses can be independently evaluated using  $G_q$  and  $S_B$  (rather than  $S_q$ ), respectively, obtained from  $\Delta$ - and  $\Psi$ -ARES;
- 2) Quantitative mapping functions were established, including  $G_q$  for SR evaluation as a function of  $\sigma$  and  $S_B$  for SSD evaluation as a function of SSD layer thickness.

#### 2.4. Principle of ARE

Variable angle spectroscopic ellipsometry (VASE) has traditionally been used to acquire ARES for SSD evaluation [30,31]. However, VASE measurements are inherently time-consuming, requiring typically 2–10 min per ARES acquisition [30], which significantly limits its use to off-line metrology. To overcome this limitation, building upon other reported works [41–45], we propose a new and effective ARES measurement method. ARE [35,46] captures polarized angle-resolved images through a high-NA (numerical aperture) objective lens’s back-focal

plane (BFP), allowing ARES to be extracted from a single or a few images with polarization modulation.

The principle of ARE is illustrated in Fig. 3. An ARE system consists of three main parts: an illuminating light source, a measuring probe, and a detection unit. A white light source is collimated and passed through a narrow-band filter and a polarizer to generate monochromatic, polarized light. The beam is directed via a beam splitter and focused onto the sample surface through a high-NA objective lens. The *s*- and *p*-polarized components of the illuminating light vary with azimuthal angles across the sample surface. Reflected light from different incident angles is spatially resolved and focused at distinct points on the Fourier pupil plane. After modulation through another polarizer, the reflected light field at the BFP is captured by a 2D detector. Azimuthal polarization modulation enables the efficient extraction of ellipsometric parameter spectra from the BFP intensity image.

The relationship between the BFP radius and the incidence angle is depicted in Fig. 3(b). The radius corresponds to the NA of the objective lens, ranging from  $-\sin^{-1}(\text{NA})$  to  $\sin^{-1}(\text{NA})$ . The relationship between the output electromagnetic wave amplitude  $E_{\text{out}}$  and the input wave amplitude is expressed as:

$$E_{\text{out}} = R_A(-A)J_A R_A(A)R(-\varphi)J_M J_S(\theta)R(\varphi) \times R_P(-P)J_P R_P(P)E_{\text{in}} \quad (3)$$

where  $J_P$ ,  $J_S$ ,  $J_A$ , and  $J_M$  are the Jones matrices of the polarizer, sample, analyzer and redirected reflection, respectively;  $R$  is the rotation matrix,  $P$  and  $A$  represent the angle of the polarizer and analyzer; and  $\varphi$  is the azimuthal angle. If  $A = \pi/4$  and  $P = 0$ , the BFP output intensity  $I_{\text{out}} = E_{\text{out}} \cdot E_{\text{out}}^*$ , has the following form:

$$I_{\text{out}}(\theta, \varphi) = \alpha_0(\theta) + \alpha_1(\theta)\sin\left(2\varphi + \frac{\pi}{4}\right) + \alpha_2(\theta)\sin(4\varphi) \quad (4)$$

which is a function of incident angle  $\theta$  and azimuthal angle  $\varphi$ . Harmonic coefficients  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are extracted using digital phase-shifting [46] from a single-shot BFP image.  $\Psi$ - and  $\Delta$ -ARES for each incident angle are calculated as:

$$\tan^2\Psi(\theta) = \frac{\sqrt{2}\alpha_0(\theta) + \alpha_1(\theta)}{-\sqrt{2}\alpha_0(\theta) + \alpha_1(\theta)} \quad (5)$$

$$\cos(\theta) = \frac{(2\alpha_2(\theta) - \alpha_0(\theta))(\tan^2\Psi(\theta) + 1)}{2\alpha_2(\theta)\tan\Psi(\theta)} \quad (6)$$

### 3. Results and discussion

#### 3.1. Experimental setup

A set of ultra-precision polished  $\text{CaF}_2$  crystals (Shanghai Puruiduo Optical Materials Co., China) were used as “ideal” samples. These samples have a diameter of 30 mm and a thickness of 2 mm. The SR of the ideal sample was measured using an atomic force microscope (AFM, Bruker Dimension Icon), as shown in Fig. 5(a) and (e). The samples were sequentially ground and then polished using a BUEHLER EcoMet 250 system to generate SSD with varying severities, as shown in Fig. 4(a). The processing parameters for SSD artefact preparation are summarized in Table 3. To meet the polishing requirements, the bottom surfaces of the  $\text{CaF}_2$  crystals were embedded in resin before processing.

Samples were cleaned using an ultrasonic cleaning machine with deionized water and ethanol and then dried with pure nitrogen gas for SSD inspection. An ARE system was developed to evaluate the surface quality of the samples, as shown in Fig. 4(b). WLI and TEM were used to characterize SR and SSD of the samples, respectively, to validate the accuracy of the ARE method. The diameter of the measured light spot was  $54.91 \mu\text{m}$ . The measurement time is directly affected by the exposure time, in this paper, the time required for a single measurement is 50 ms. Focused ion beam (FIB) technique was used to prepare cross-sectional samples for TEM observations.

#### 3.2. Results of AFM, WLI and TEM

Typical samples were selected for SR measurement using a Zygo NewView 6000, a powerful WLI. The effect of the polishing process on SR was investigated by observing samples at three stages: the initial stage of rough polishing (R20), the final stage of rough polishing (R70), and the final stage of fine polishing (F30). SR measurements were performed on randomly selected areas (three repetitions) for each sample using a 20X objective lens. Surface topography portions with  $850 \mu\text{m}$  and  $120 \mu\text{m}$  side lengths were obtained.

Fig. 5 presents the WLI surface topography results with local magnification.

1) After 20 min of rough polishing, surface defects such as holes and cracks were observed, indicating significant surface irregularities.

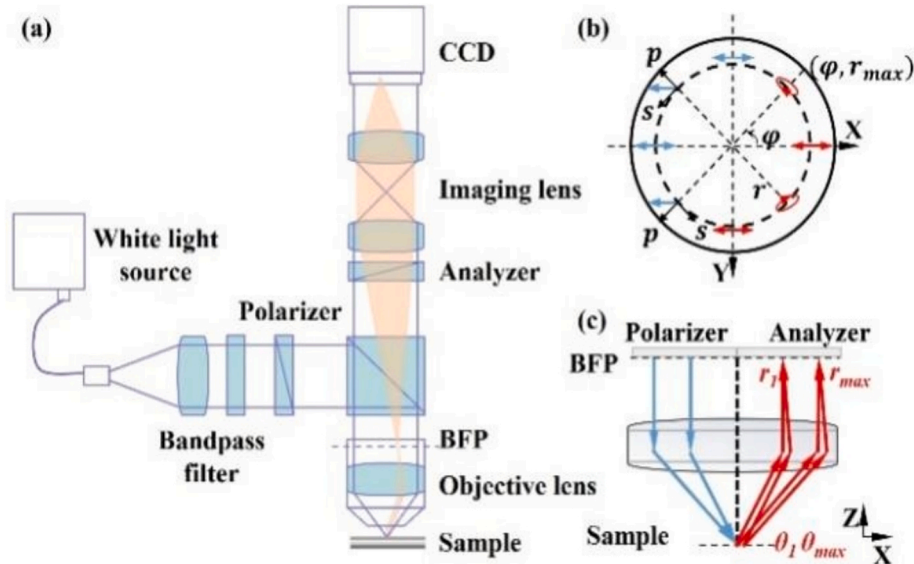


Fig. 3. Principle of ARE measurement. (a) Optical configuration, (b) polarization status on the BFP pupil, and (c) the relationship between the BFP radius ( $r$ ) and incident angle ( $\theta$ ). Incident and reflected beams are noted in blue and red, respectively.

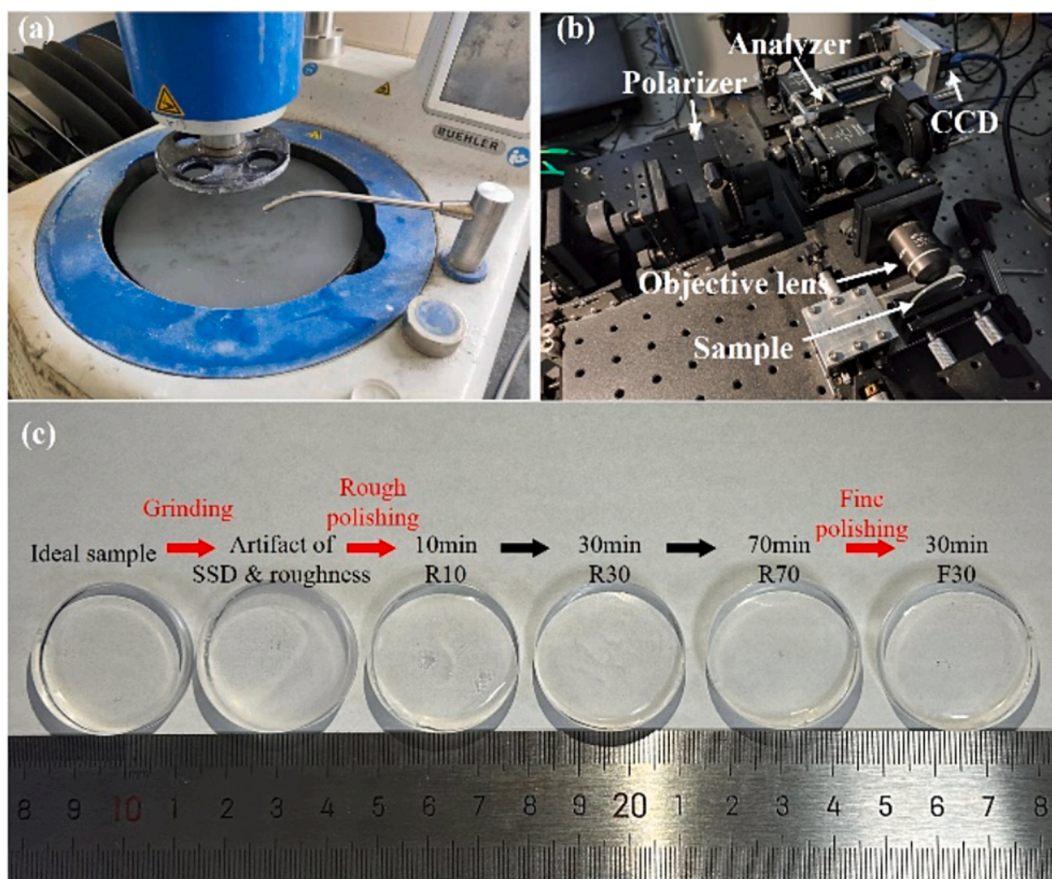


Fig. 4. Experimental equipment and materials. (a) Polishing machine, (b) ARE system, and (c) typical processed samples. R and F represent the rough and fine polishing stages, respectively.

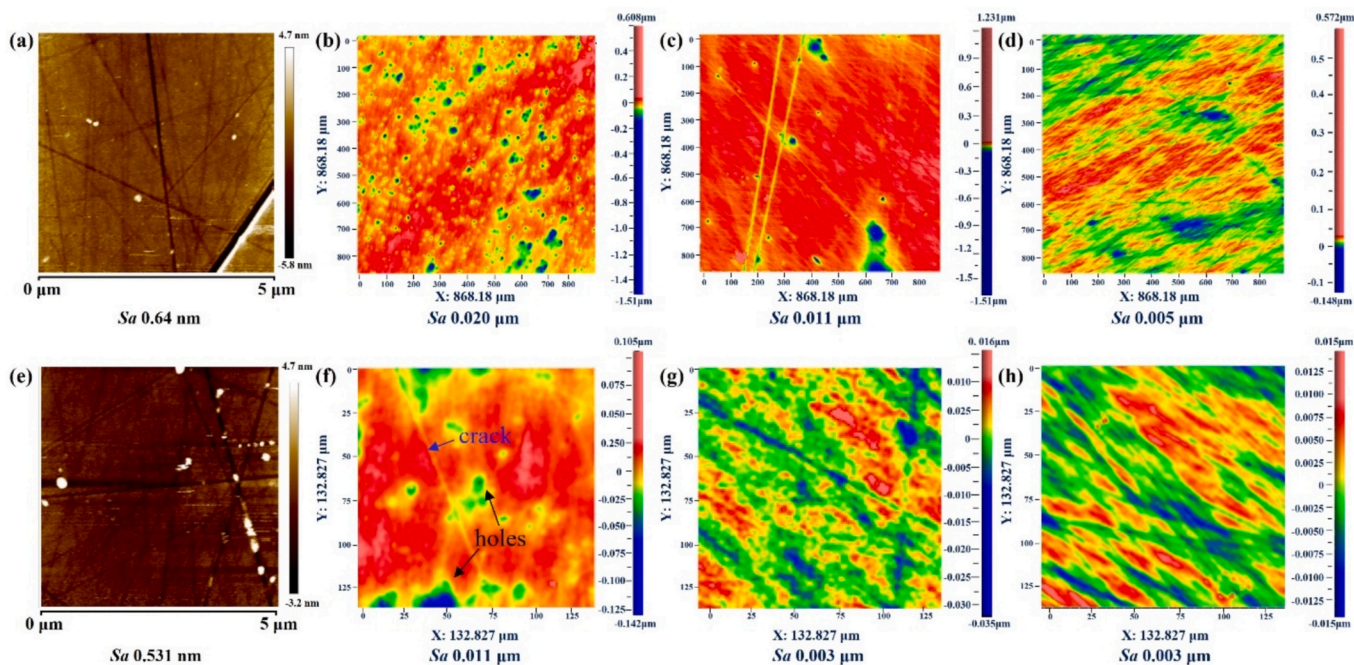


Fig. 5. AFM and WLI measurement results of  $\text{CaF}_2$  samples. (a, e) Surface topography of “ideal” sample; (b-d) surface topography of R20, R70, F30 and their local zooms (f-h), with a mean  $Sa$  of 0.5 nm, 10.6 nm, 3.4 nm and 2.6 nm, respectively.

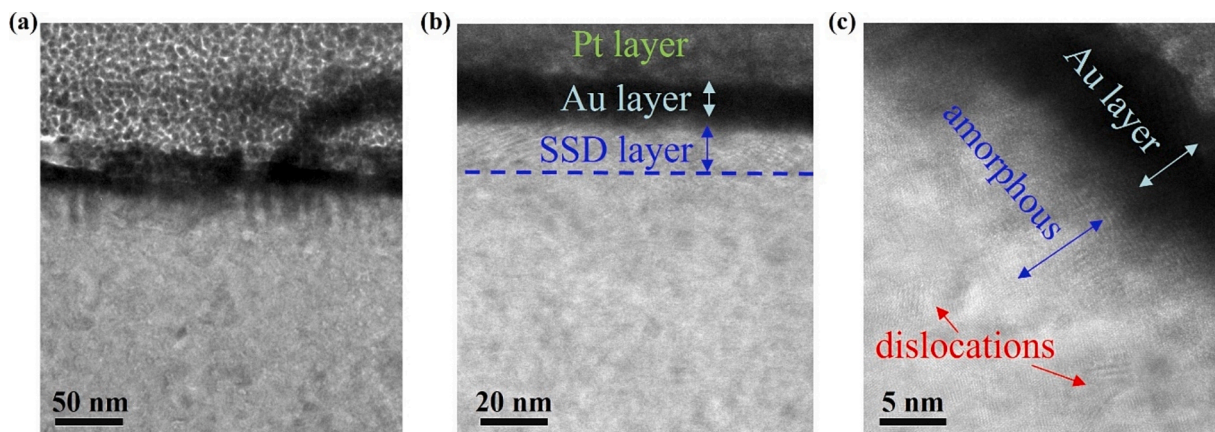
**Table 3**  
Processing parameters of SR-SSD artifacts through grinding and polishing.

Sample	Process	Material	Parameter	Duration
SR/SSD Artifact	Rough grinding	Grinding disc: W40 Cast iron plate	Polishing speed: 50 r/ min Grinding disc speed: 50 r/ min Grinding load: 5 N/cm <sup>2</sup>	4 min
	Fine grinding	Grinding disc: W20 Cast iron plate	Polishing speed: 50 r/ min Grinding disc speed: 60 r/ min Grinding load: 5 N/cm <sup>2</sup>	12 min
R10	Rough polishing	Polishing disc: IC 1000	Polishing speed: 50 r/ min	10 min
R20		polyurethane plate	Polishing speed: 50 r/ min	20 min
R30		Polishing solution: W2.5	Polishing disc speed: 70 r/ min	30 min
R50		Al <sub>2</sub> O <sub>3</sub>	Polishing disc speed: 70 r/ min	50 min
R70			Polishing load: 5 N/cm <sup>2</sup>	70 min
F10	Fine polishing	Polishing disc: IC 1000	Polishing speed: 50 r/ min	10 min
F30		polyurethane plate Polishing solution: W0.1 SiO <sub>2</sub>	Polishing disc speed: 70 r/ min Polishing load: 5 N/cm <sup>2</sup>	30 min

- After 70 min of rough polishing, surface pits decrease dramatically, leaving nanoscale scratches visible, as shown in Fig. 5(c, g);
- After 30 min of fine polishing, surface defects were nearly eliminated, and the average SR,  $S_a$ , decreased to 2.6 nm, as shown in Fig. 5(d, h).

These results demonstrate that fine polishing can produce a surface suitable for next-generation atomic-level ultraprecision manufacturing.

Fig. 6 shows the cross-sectional TEM (FEI Tecnai G2 20) images of sample F30 at different resolutions. Pt and Au layers were sputtered on sample surfaces to prevent FIB damage and enhance conductivity. The results show that the SSD layer thickness is approximately 20 nm, with an amorphous layer thickness of ~10 nm. Lattice dislocations were found below the layer. These observations are consistent with previous studies [30], which reported the formation of amorphous layers during conventional grinding and polishing. Optimized processing methods,



**Fig. 6.** TEM images of SSD of sample F30 with different resolutions.

such as plasma treatment with chemical mechanical polishing [47], inductively coupled plasma [48], and plasma-assisted polishing [49], have been developed to produce SSD-free polished crystals.

### 3.3. Results of ARE

Fig. 7 presents the ARES of  $\Psi$  and  $\Delta$  obtained from the CaF<sub>2</sub> samples using the proposed ARE system. Ten randomly selected surface portions of each sample were measured and averaged to ensure reproducibility. Fig. 7(e) shows the ten original  $\Psi$ -ARES curves of sample R70. During the evaluation of SR and SSD, a fifth-order polynomial was used to fit the ten raw spectra (within the range of 40–60°) for noise suppression, as demonstrated in Fig. 7(a) and (d). Fig. 7(a) shows that the mean  $\Delta$ -ARES of the treated samples gradually approached the ideal sample along with the progression of the polishing process.

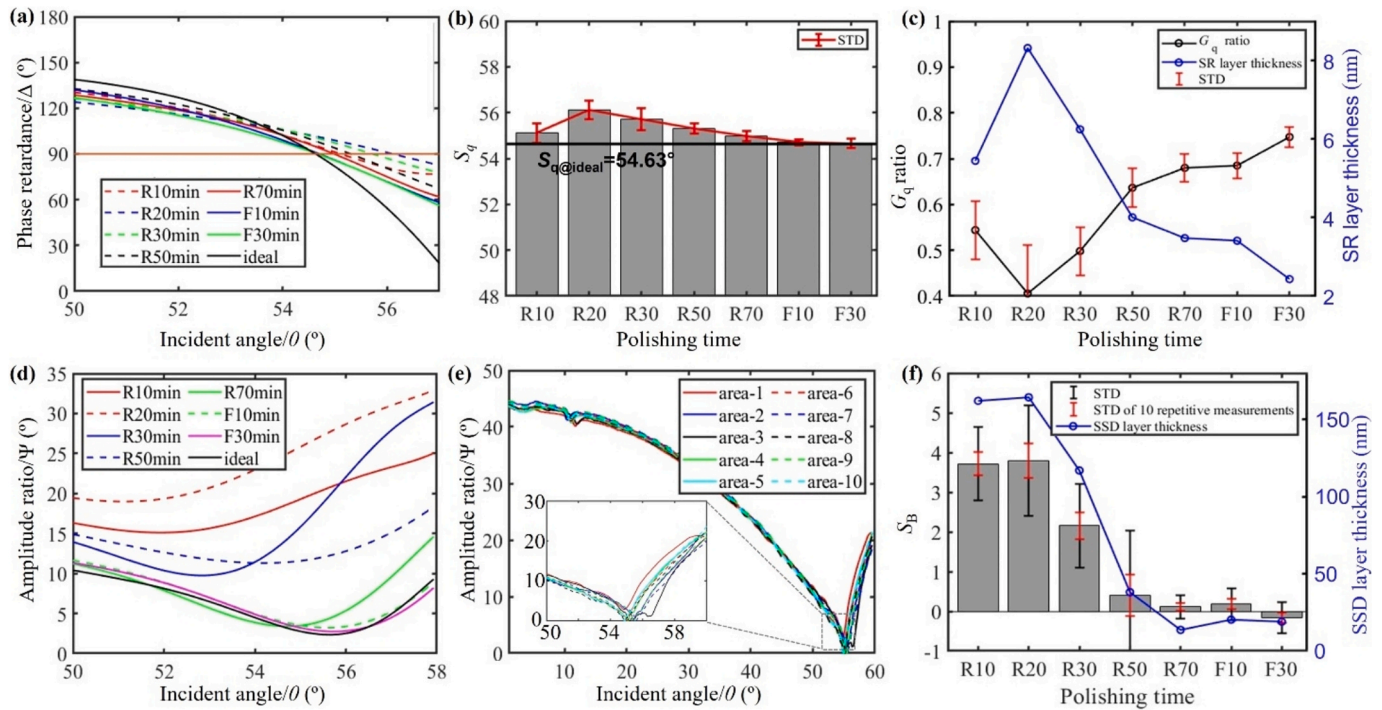
Fig. 7(b) and (c) present the variations of  $S_q$  and the  $G_q$  ratio during different processing stages based on  $\Delta$ -ARES analysis. Key findings include:

- During early polishing (R10-R20),  $S_q$  and  $G_q$  (ratio over that of the ideal sample was analyzed) increased due to the exposure of damages caused by grinding of the SSD artefacts. This phenomenon aligns with previous findings by Chu et al. [15], who observed similar non-monotonic SR variations using chemical etching.
- After extended polishing (R50-R70),  $G_q$  and  $S_q$  returned gradually to a constant after 50 min of rough polishing, indicating the achievement of a stable surface quality.
- After final polishing (F10-F30),  $G_q$  indicated that SR approached about 2 nm, consistent with the WLI results. This correlation demonstrates the validity of  $G_q$  for SR evaluation, even though  $G_q$  was derived through FDTD simulations without considering SSD presence.

However, the TEM results revealed that the subsurface contained an amorphous layer and lattice dislocations. This contradicts  $S_q$ , which converged to that of the initial “ideal” sample, confirming that  $S_q$  alone is unreliable for SSD characterization (consistent with the conclusion from the EMA study in Fig. 1(b)).

The  $\Psi$ -ARES method was employed for SSD evaluation, as shown in Fig. 7(d). As the polishing process progressed, the  $\Psi$ -ARES spectra increasingly resembled those of the ideal sample. The following key insights are resulted from Fig. 7(f):

- During early polishing (R10-R20),  $S_B$  remained unchanged between R10 and R20, indicating that the polishing had not removed SSD.
- After extended polishing (R50-R70),  $S_B$  approaches the ideal sample, but a larger STD indicates a non-uniform SSD removed.



**Fig. 7.** ARES analysis for SR and SSD evaluation of CaF<sub>2</sub> samples under various processing conditions. (a) Zoomed view of fitted  $\Delta$ -ARES and corresponding  $S_q$  (b) and  $G_q$  for SR evaluation (c), where  $G_q$  ratio over that of the ideal sample is analyzed for convenience (same in Fig. 2c), and the error bars represent the STD of 10 measurement areas; (d) zoomed view of fitted  $\Psi$ -ARES and extracted  $S_\beta$  for SSD evaluation (f); (e) original and zoom view  $\Psi$ -ARES of ten repetitive measurements for sample R70.

3) After final polishing (F10-F30),  $S_\beta$  and STD stabilized to nearly match the ideal sample. Combined with  $G_q$  results in Fig. 7(c), this indicates the near-complete elimination of SSD.

The accuracy of the  $S_\beta$  method was validated by comparing SSD values derived from  $S_\beta$  at F30 with those from TEM images. In addition, Fig. 7(f) confirmed a slight improvement in subsurface quality due to the over-polishing process, consistent with the observations by Yao et al. [30]. These results demonstrate the feasibility of the  $S_\beta$  method for evaluating SSD and highlight the reliability of the ARE system for characterizing complex samples.

It should be noted that, at the initial stage of rough polishing, the STDs of  $S_\beta$  for SSD measurements were significant due to large SR and surface non-uniformity. However, repetitive measurements (10 repetitions in the study) ensured acceptable reliability. The single-shot nature of ARE technology facilitates such repeated measurements without compromising efficiency, further enhancing the robustness of the proposed method.

#### 4. Conclusion

This study introduces a novel method for evaluating the surface quality of polished crystals using angle-resolved ellipsometry. Addressing the limitations of the current quasi-Brewster angle technique, we proposed a new approach based on angle-resolved ellipsometric spectra (ARES) of  $\Psi$  for distinguishing subsurface damage (SSD) from surface roughness (SR).

Through finite-difference time-domain simulations, we demonstrated the feasibility of  $\Psi$ -ARES under oblique incidence conditions, establishing quantitative relationships between SR layer thickness and gradient angle  $G_q$ , as well as SSD layer thickness and Brewster angle shift  $S_\beta$ , obtained from  $\Delta$ - and  $\Psi$ -ARES, respectively. These insights provide a robust theoretical foundation for nondestructive SSD evaluation.

Experimental validation was conducted on CaF<sub>2</sub> crystal samples with varying polishing parameters. The measurement results obtained from a

developed ARE system were validated using white light interferometry and transmission electron microscopy, confirming the reliability of the  $\Psi$ -ARES-enhanced q-BAT method. The results also highlighted the capability of  $\Psi$ -ARES to assess both SR and SSD independently, offering a significant advantage over existing methods.

This work demonstrates that the  $\Psi$ -ARES-enhanced q-BAT method, integrated with snapshot ARE, provides a rapid, nondestructive, and highly accurate tool for surface quality evaluation. Though CaF<sub>2</sub> crystals are demonstrated only in the study, its application extends beyond polished to other materials in ultra-precision machining, making it a promising technology for advancing quality control and metrology in manufacturing processes.

#### CRediT authorship contribution statement

**Tianqi Jia:** Writing – original draft, Methodology, Investigation, Data curation. **Jian Wang:** Writing – review & editing, Validation, Supervision. **Lihua Peng:** Writing – review & editing, Writing – original draft, Methodology. **Wei Guo:** Writing – review & editing, Validation. **Wenjuan Sun:** Writing – review & editing, Validation. **Shiyuan Liu:** Writing – review & editing, Supervision.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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## Data availability

Data will be made available on request.

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